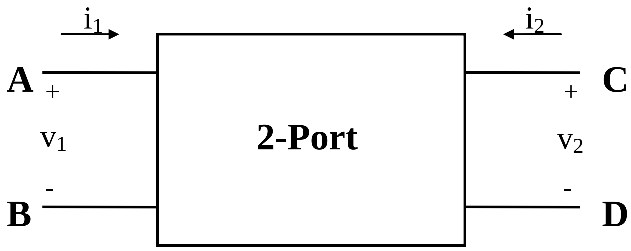
ECE 521 Fall 2016

Homework #1 - Part 1 (Due Oct. 12)

Note:
Problems 3-5 are from Prof. Jacob White of MIT.

1. The 2-port shown below is defined in terms of its *h*-parameters

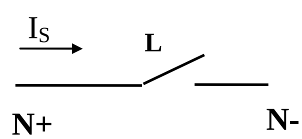


The *h*-parameters are defined as

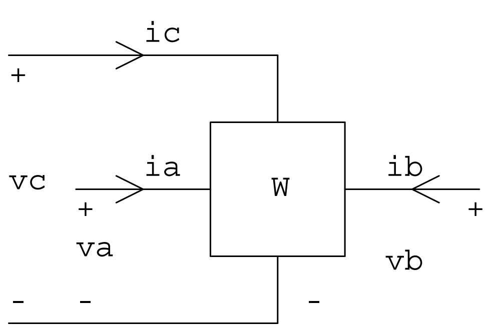
$$v_1 = h_{11} i_1 + h_{12} v_2$$
$$i_2 = h_{21} i_1 + h_{22} v_2$$

a) Write the MNA stamp for this element.
b) When can a plain nodal analysis be used for this element? Write the nodal stamp.

2. The boolean switch shown below is controlled by the logic variable L. The switch is closed when $L=1$ and open when $L=0$. Write the MNA stamp that is valid for the switch in both on and off conditions. Note the stamp will be a function of the values of L.



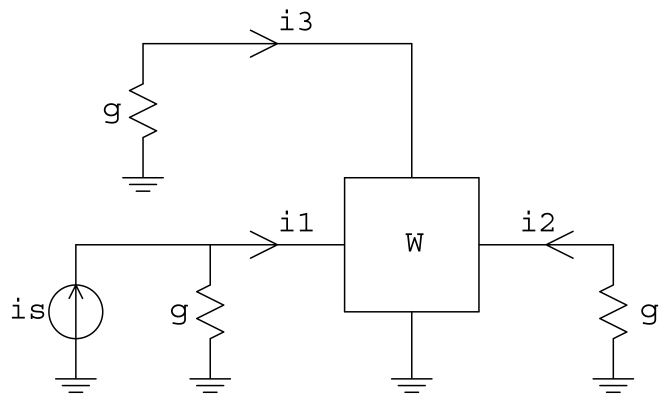
3. Consider a 4-terminal element



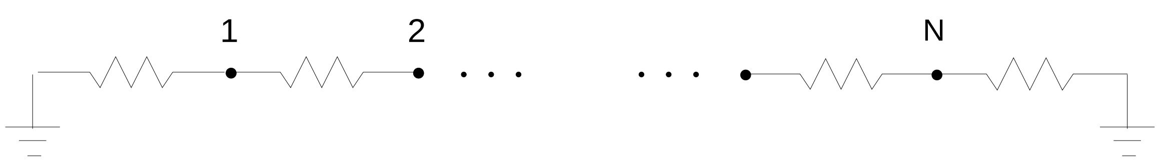
described by the constitutive relation:

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} + \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} = 0$$

a) Explain why you can **not** use plain nodal analysis for circuits with such an element.
b) Using any equation formulation approach you wish, generate the set of equations for the following circuit.



4. Consider the following line of *N* resistors



a) Determine the number of nonzero entries in the $N \times N$ conductance matrix associated with a *N*-long line of identical resistors of value *R*.
b) Determine the number of nonzero entries in the $N \times N$ resistance matrix associated with the *N*-long line of resistors. Recall that the resistance matrix is the inverse of the conductance matrix.
c) Determine the number of nonzero entries in the *L* and *U* factors of the conductance matrix associated with a *N*-long line of resistors. Compare to your result from part (b) for $N = 1000$
d) What is the moral of this exercise?

5. Consider $A \in R^{N \times N}$ whose only nonzero elements are given by the following:

$$A_{i,i} = 1 \qquad\qquad i \in \{1, \ldots, N\}$$
$$A_{i+1,i} = -\alpha \qquad\qquad i \in \{1, \ldots, N-1\}$$
$$A_{1,N} = -\alpha$$

a) Suppose β is the largest number your computer can represent without overflowing, and assume $\alpha > 1$. Give a formula in terms of α and β for the largest value of N for which A can be factored without overflowing, using Gaussian elimination with no reordering.

b) Give a reordered Gaussian elimination algorithm for solving the above matrix which will not cause an overflow, regardless of the size of N.