Homework #2 - Part 1 due October 26

1) Consider the linear system $Ax = b$, where

$$A = \begin{bmatrix} 10^{-5} & 10^{-5} & 1 \\ 10^{-5} & -10^{-5} & 1 \\ 1 & 1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 2 \times 10^{-5} \\ -2 \times 10^{-5} \\ 1 \end{bmatrix}$$

(a) Write the exact solution of the above system

(b) Solve the system using three-digit floating-point arithmetic with partial pivoting.

(c) Solve the system using three-digit floating-point arithmetic with complete pivoting.

2) Consider the following matrix

$$A = \begin{bmatrix} a_{11} & & a_{13} & a_{14} & \\ & a_{22} & & & a_{25} \\ a_{31} & & a_{33} & & a_{35} \\ a_{41} & & & a_{44} & \\ & & a_{53} & & a_{55} \end{bmatrix}$$

Assuming that the pivots are restricted to the main diagonal, find the pivoting order if Markowitz reordering is used. How many fill-ins are created?

3(a) Show that the reciprocal of any real number $n > 0$ can be found from the recursive relation $x_{k+1} = x_k(2 - nx_k)$. Suppose the number is π, calculate x_4 for $x_0 = 0.1$, and $x_0 = 0.7$. Comment on your results.

(b) Suppose the Newton-Raphson method is applied to solve the system of equations $Ax = b$. What is the Jacobian matrix? In how many iterations does the method converge? What should x_0 be for the method to converge?

4) Write code (in your favorite language) that will find the solution of a nonlinear equation in one unknown using Newton's method. To make it a generic procedure the Newton loop should make calls to two functions FX and DX that compute the function and derivative values. Assume $\epsilon_a = 1 \times 10^{-6}, \epsilon_r = 1 \times 10^{-3}$. Plot FX and $||x^k - x^*||$ as a function of the iteration count for the following cases:

(a) $f(x) = x^2 - 1$, $x^0 = 0.1, 0.5, 0.9$

(b) $f(x) = x^2$, $x^0 = 1, 0.5, 0.1$

(c) $f(x) = 10^{-3} - 10^{-16}(e^{40x} - 1)$, $x^0 = 0.3, 0.5, 0.7, 0.8$

(d) $f(x)$ as in (a) but using the following function as the derivative of $f(x)$: $f'(x) = (f(x+1) - f(x))$. Compare your results with that of (a).