

Notes

1. Problem 3 is from Prof. Jacob White of MIT (Course 6.336).

1) Consider the MOSFET model as a drain current described by the general form $I_{DS} = f(V_{GS}, V_{DS}, V_{BS})$, and linear capacitances $C_{GD}, C_{GS}, C_{DB}, C_{SB}, C_{GB}$

a) Write the stamp (matrix and RHS) for a Newton iteration of dc analysis.

b) Write the stamp (matrix and RHS) for small-signal AC Analysis.

2) A linear multistep integration formula can be cast in a general form $\dot{x}_n = \alpha x_n + \beta$ where α depends on the stepsize h and β is a function of the x values at the previous time points.

a) What are α and β for BE, TR and Gear-2 methods?

b) Using the general form above write the companion models and stamps for a linear capacitor and inductor.

c) Consider a nonlinear capacitor described by $q = q(v)$. Draw the companion model for this capacitor for timepoint n . Assuming a Newton method is used to solve the nonlinear equations, what is the companion model at iteration $k + 1$?

3) Consider using the following integration method to solve for $x(t)$ which satisfies $\dot{x}(t) = \lambda x(t)$,

$$\frac{x_n - x_{n-2}}{2h} = \lambda x_{n-1},$$

where $x_0 = 1$. Note that x_n approximates $x(t)$ at time point $t = nh$.

a) Determine the Local Error (LE) of this “leap-frog” method.

b) Is the method stable? Will the method converge?

c) Plot and compare the computed and the exact solution for the case $\lambda = -1$, and on the interval $t \in [0, 10]$. Use $h = 2$, $h = 0.5$, and $h = 0.1$.

d) Look carefully at your plots, and explain your results in part c.

4) A linear multistep integration formula is described by

$$x_n - \alpha_1 x_{n-1} - h\beta_0 \dot{x}_n - h\beta_1 \dot{x}_{n-2} = 0$$

a) Determine the coefficients so that this is a second-order method.

b) What is the Local Error (LE) for this method?

c) Determine the Γ_σ contour and use it to find the region of absolute stability.