ECE 521 Lecture # 6 Oct. 10, 2016

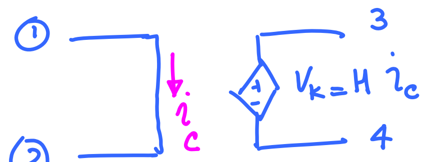
Note Title 10/10/2016

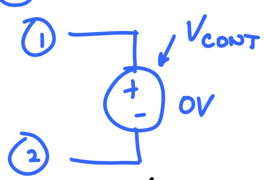
HW # 1 Due Wed Oct. 12

Current controlled source

netlist

HK 3 4 VCONT H

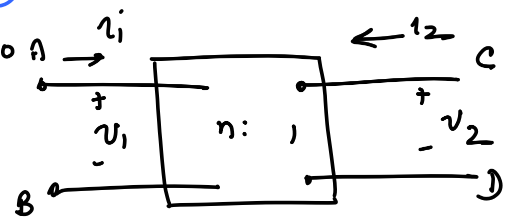




Transformer

$v_1 = n\, v_2$: $(V_A - V_B) - n(V_C - V_D) = 0$

$i_1 = -\frac{1}{n} i_2$



	V_A	V_B	V_C	V_D		I_2
A					1	$-\frac{1}{n}$
B						$+\frac{1}{n}$
C						1
D					1	-1
BCR						

BCR [1 -1 -n +n]

Gyrator.

Nonlinear equation solution

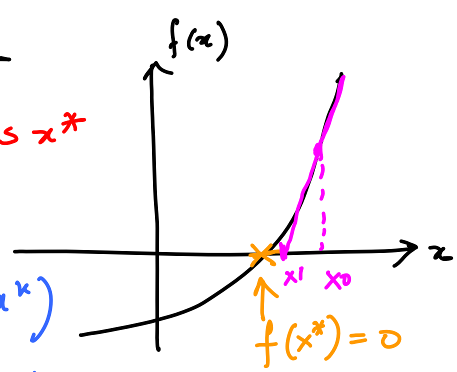
$f(x) = 0$; solution is x^*

$$f'(x^k)(x^{k+1} - x^k) = -f(x^k)$$

$$x^{k+1} - x^k = -\left[f'(x^k)\right]^{-1} f(x^k)$$

$$x^{k+1} = x^k - \left[f'(x^k)\right]^{-1} f(x^k)$$

$$f'(x^k) = \left.\frac{\partial f}{\partial x}\right|_{x^k}$$

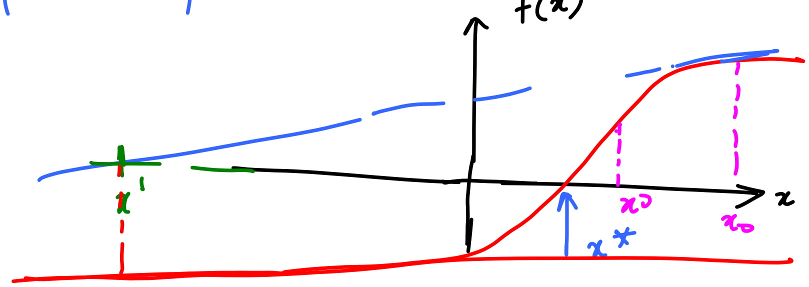


$\frac{\partial f}{\partial x}$ is bounded away from zero

$\frac{\partial^2 f}{\partial x^2}$ is bounded if x^0 was sufficiently close to x^*

Then Newton's method converges to the solution and the convergence rate is quadratic

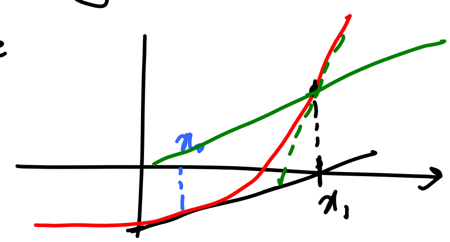
$$|x^{k+1} - x^*| \leqslant c\,|x^k - x^*|^2$$



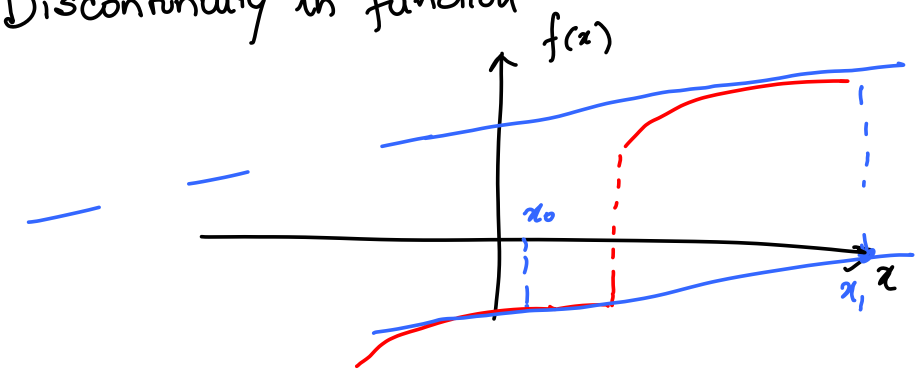
Potential problems:

1) x^0 is not close enough to x^*

2) Error in derivative



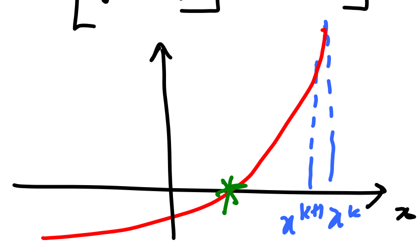
3) Discontinuity in function



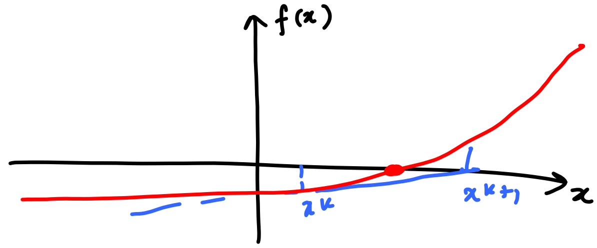
Convergence checks

$$(x^{k+1} - x^k) = -\left[f'(x^k)\right]^{-1} f(x^k)$$

$\|x^{k+1} - x^k\| <$ tolerance



$\|f(x)\| <$ tolerance



Multidimensional case

$$\text{KCL equations} \left\{ \begin{array}{l} f_1(x_1, x_2, \cdots x_n) = 0 \\ \vdots \\ f_n(x_1, x_2, \cdots x_n) = 0 \end{array} \right.$$

$$J(\underline{x}^k)(\underline{x}^{k+1} - \underline{x}^k) = -\underline{f}(\underline{x}^k)$$

$$J(x^k) = \left.\begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \vdots & & & \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix}\right|_{x^k}$$

Computational Procedure

1) Evaluate $f(x^k)$ and $J(x^k)$ ←

2) Assemble circuit equations

$$J(x^k)\underbrace{(x^{k+1} - x^k)}_{\Delta x^{k+1}} = -f(x^k)$$

OR

$$J(x^k)\, x^{k+1} = J(x^k)\, x^k - f(x^k)$$

$$A x = b \quad \text{linear system of equations}$$

3) Do not compute the inverse of $J(x^k)$ instead solve the linear equations using LU factorization & F/B solves (+ sparse matrix techniques)

4) Solution gives x^{k+1} (or Δx^{k+1})

5) Repeat procedure until convergence or exceeds Maximum # of iterations (default is SPICE 100)

Remarks:

- Since the device equations are known, the derivatives can be computed explicitly (or as equations)
- Automatic differentiation
- The significant computational cost/iteration

  $J(x^k)$, $f(x^k)$ ← model evaluation

  Solve $Ax = b$ ← linear eqn. solution

  This can be a significant cost in very complex models

- Some versions of SPICE will skip model evaluation at every iteration (BYPASS = 1)

Application to circuits:

$$f(x) = x_1 + x_2 + x_3 = 0$$

Linearize per Newton's method

$$\left.\frac{\partial f}{\partial x}\right|_{x^k} \left(x^{k+1} - x^k\right) = -f(x^k)$$

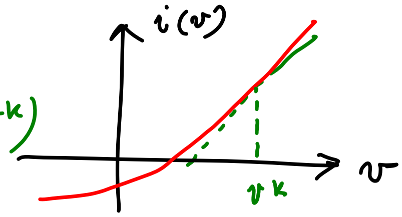
$$\left.\frac{\partial f}{\partial x}\right|_{x^k} x^{k+1} = \left.\frac{\partial f}{\partial x}\right|_{x^k} x^k - f(x^k)$$

Linearization of linear equations → no change

What about a nonlinear resistor?

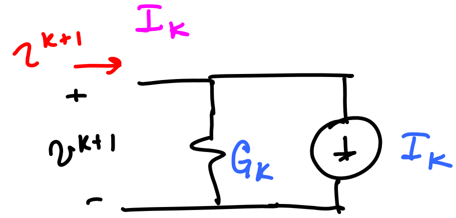
$$i = i(v)$$

$$i^{k+1} = i^k + \underbrace{\frac{\partial i}{\partial v}\Big|_{v^k}}_{G_k} (v^{k+1} - v^k)$$

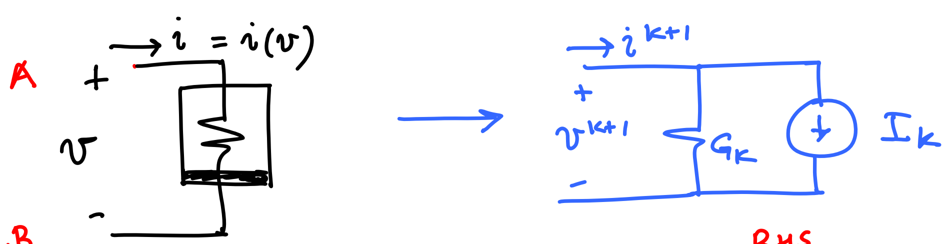


$$i^{k+1} = G_k (v^{k+1} - v^k) + i^k$$

$$= \underbrace{G_k}\, v^{k+1} + \underbrace{i^k - G_k v^k}_{I_k}$$



Companion network (or model)



Stamp

$$\underbrace{\begin{array}{c|cc} & V_A & V_B \\ \hline A & +G_k & -G_k \\ B & -G_k & +G_k \end{array}}_{J^k} \qquad \underbrace{\begin{array}{c} \text{RHS} \\ \hline -I_k \\ +I_k \end{array}}_{f(x^k)}$$

Diode

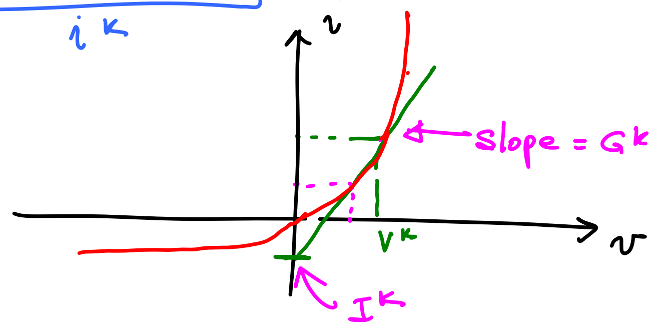
$$i = I_s\left(e^{v/V_{th}} - 1\right)$$

I_s and V_{th} are constants at a given temp.

$$G_k \rightarrow \frac{\partial i}{\partial v}\Big|_{v^k} \qquad I_k \rightarrow i^k - G_k v^k$$

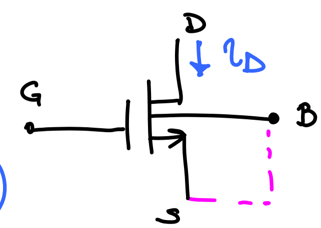
$$G_k = \frac{I_s}{V_{th}} e^{v/V_{th}} \Big|_{v = v^k}$$

$$I_k = \underbrace{I_s\left(e^{v^k/V_{th}} - 1\right)}_{i^k} - G_k v^k$$



MOSFET

V_{GS}, V_{DS}



In saturation:

$$i_D = \frac{K'}{2}\frac{W}{L}\left(V_{GS} - V_T\right)^2\left(1 + \lambda V_{DS}\right)$$

$\hookrightarrow$ Level-1 MOS model

$$i_D = f(V_{GS}, V_{DS})$$

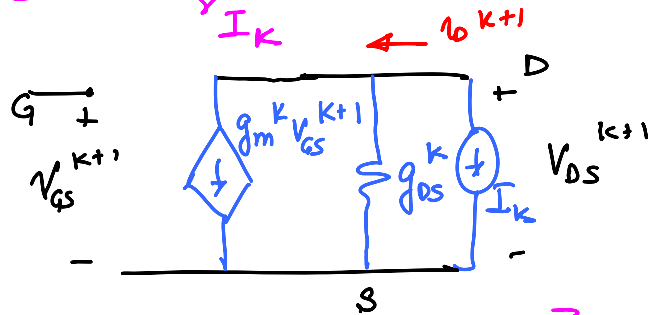
$$i_D^{k+1} = i_D^k + \overbrace{\frac{\partial i_D}{\partial V_{GS}}\Big|_{V_{GS}^k, V_{DS}^k}}^{g_m^k}\left(V_{GS}^{k+1} - V_{GS}^k\right)$$
$$+ \underbrace{\frac{\partial i_D}{\partial V_{DS}}\Big|_{V_{DS}^k, V_{GS}^k}}_{g_{DS}^k}\left(V_{DS}^{k+1} - V_{DS}^k\right)$$

$$\rightarrow i_D^{k+1} = i_D^k + g_m^k\left(V_{GS}^{k+1} - V_{GS}^k\right) + g_{DS}^k\left(V_{DS}^{k+1} - V_{DS}^k\right)$$

$$g_m^k = K'\frac{W}{L}\left(V_{GS} - V_T\right)\left(1 + \lambda V_{DS}\right)\Big|_{V_{GS}^k, V_{DS}^k}$$

$$g_{DS}^k = \frac{K'}{2}\frac{W}{L}\left(V_{GS} - V_T\right)^2 \lambda \Big|_{V_{GS}^k, V_{DS}^k}$$

$$i_D^{k+1} = g_m^k V_{GS}^{k+1} + g_{DS}^k V_{DS}^{k+1} + \underbrace{i_D^k - g_m^k V_{GS}^k - g_{DS}^k V_{DS}^k}_{I_K}$$

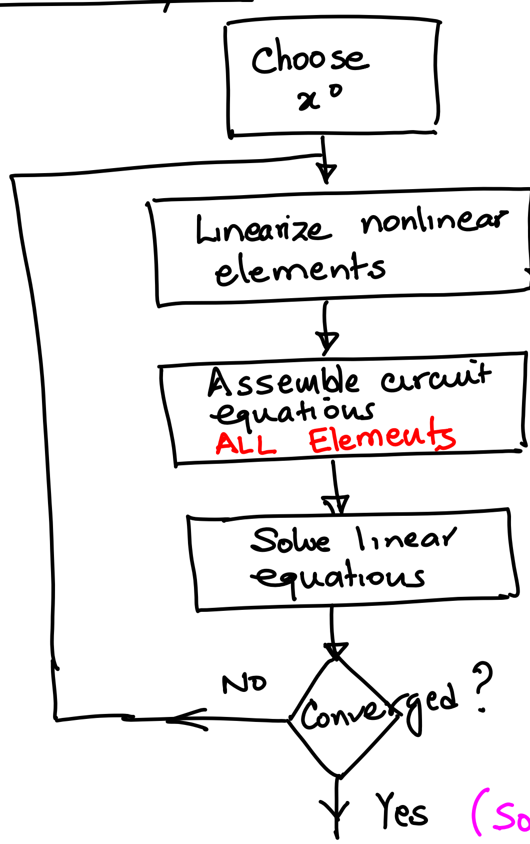


Companion network for MOSFET

Stamp

$$\begin{array}{c|ccc}
 & V_D & V_S & V_G \\
\hline
D & +g_{DS}^k & -g_{DS}^k - g_m^k & +g_m^k \\
S & -g_{DS}^k & +g_{DS}^k + g_m^k & -g_m^k \\
G & 0 & 0 & 0
\end{array}
\qquad
\text{RHS} \begin{bmatrix} -I_K \\ +I_K \\ 0 \end{bmatrix}$$

SPICE DC analysis



Compute G_K, I_K

Stamp $J(x^k)$, $f(x^k)$

$J^k V^{k+1} = J^k V^k - f(V^k)$

or exceed maximum iteration

(Solution) or non convergence

SPICE Convergence Checks

For each node:

$$|V^{k+1} - V^{k}| \le \underset{\uparrow \atop \text{VNTOL}}{\epsilon_A} + \underset{\uparrow \atop \text{RELTOL}}{\epsilon_R} \, \text{MAX}\left(|V^{k}|, |V^{k+1}|\right)$$

Default values: VNTOL $= 10^{-6}$ ($1 \mu V$)

RELTOL $= 10^{-3}$

SPICE doesn't do a convergence check on $|f(x)|$

Instead:

$$|I_k - \hat{I}_k| \le \underset{\uparrow \atop \text{ABSTOL}}{\epsilon_A} + \epsilon_R \, \text{MAX}\left(|I_k|, |\hat{I}_k|\right)$$

Default value of ABSTOL $= 10^{-12}$ ($1 pA$)

I_k = actual current for voltage V^k

$\hat{I}_k$ = linearized terminal current for V^k

numerical overflow issues

Limiting methods!