ECE 521 Lecture # 16 Nov. 14, 2016

Note Title 11/14/2016

HW # 3 – Part 1 Due today

HW # 4 – part 2 Due wednesday

Project proposals " Monday Nov. 21

Graded HW # 3 – Part 2 handed back

Sample exam posted on class website

Exam Mon Nov. 21

(more on Wednesday)

int *icheck;

pnjlim (*icheck)

double pnjlim(– – – icheck)

int * icheck;

double pnjlim();

Vnew = pnjlim( );

BJT

pnjlim (V_{BE}

pnjlim (V_{BC}

out of bound arrays;

$$I_c^{k} = f(v^k) - g_{mF} V_E - - -$$

$$= I_c - g_{mF} V_{be} + \frac{g_{mr}}{\alpha_R} V_{bc}$$

$$I_E^{k} = I_E + \frac{g_{mF}}{\alpha_F} V_{be} - g_{mr} V_{bc}$$

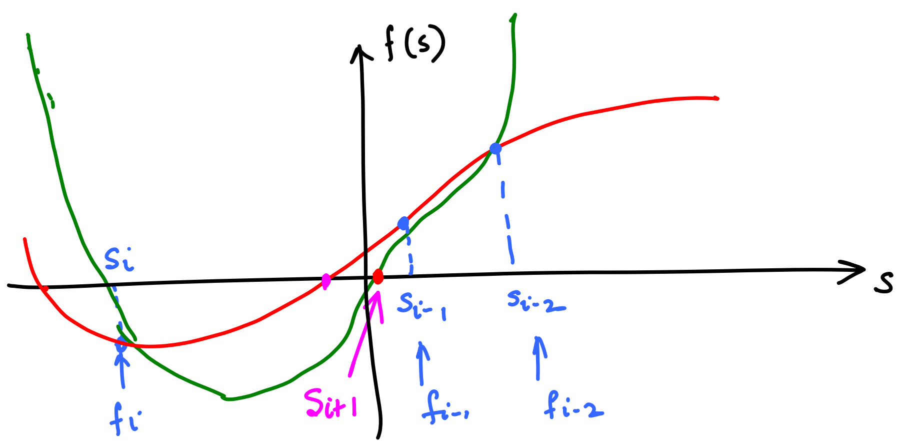
Pole/zero analysis

$\det(G + sC) = 0 \longrightarrow$ poles

$\det(G_a + sC_a) = 0 \longrightarrow$ zeros

Finding the roots of polynomials

Muller's method $\quad F(s) = a_0 + a_1 s + a_2 s^2 + \cdots + a_n s^n$



$b_0 + b_1 s + b_2 s^2$

Note that this process relies on calculating $f(s_i)$ which is $\det(G + s_i C)$. $\qquad s_i = \sigma_i + j\omega_i$

$$A = LU$$

$$\det(A) = \det(LU) = \det(L)\det(U)$$

$$L = \begin{bmatrix} l_{11} & & & 0 \\ l_{21} & l_{22} & & \\ \vdots & & \ddots & \\ l_{n1} & \cdots & \cdots & l_{nn} \end{bmatrix} \qquad U = \begin{bmatrix} 1 & u_{12} & \cdots & u_{1n} \\ & 1 & & \vdots \\ & & \ddots & u_{n-1,n} \\ & & & 1 \end{bmatrix}$$

$\det(L) = l_{11} l_{22} \cdots l_{nn}$ $\qquad$ $\det(U) = 1$

$$\det(G + s_i C) = l_{11} l_{22} \cdots l_{nn}$$

→ In the simulator a circuit load (small-signal) is performed for a value of $s = s_i$

→ LU factor the circuit matrix

Remarks

- SPICE2 doesn't support .pz analysis
- SPICE3 uses Muller's method
  HSPICE
- Muller's method is not very robust
  HSPICE has many options!

QZ algorithm $\qquad$ standard algorithm

Solves the generalized eigenvalue problem

$Ax = \lambda x$ $\quad$ eigenvalue problem

$Ax = \lambda B x$ $\quad$ generalized "

A: matrix $\quad$ B: matrix

$$Ax - \lambda B x = 0$$

$$\det(A - \lambda B) = 0$$

$$\det(G + sC) = 0$$

QZ algorithm is a dense matrix algorithm

⇒ cannot be used for circuits with > 500 nodes

What do we do for larger circuits?

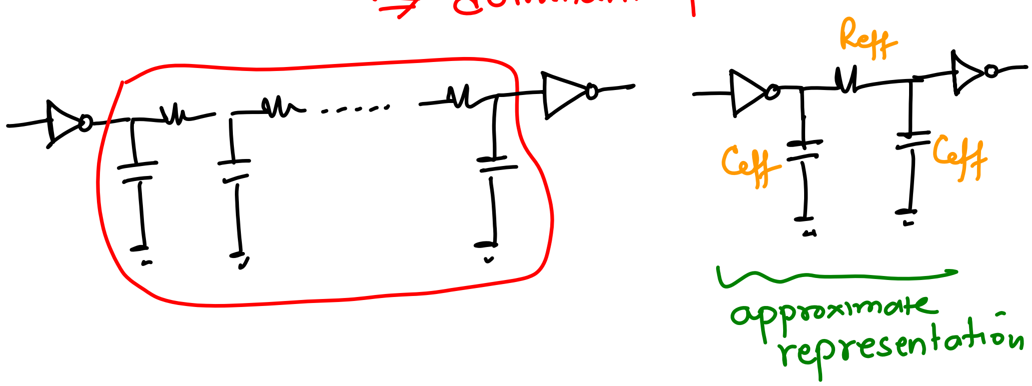
Q1 : Do we really need all of the poles of a circuit 10000 nodes $\sim$ 10 000 poles

Q2: Even if we could determine all the poles then what does one do with that info?

<u>Observation</u>

only a small number of poles are important for characterizing circuit performance

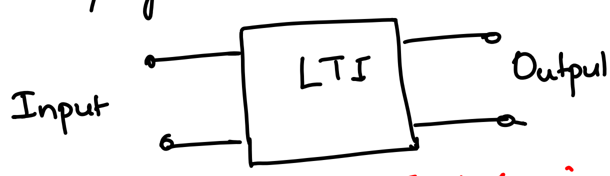
$\Rightarrow$ dominant poles



$\rightarrow$ Asymptotic Waveform Evaluation (AWE)

Pade via Lanczos (PVL)

The underlying idea is that of Padé approx.



The input/output relationship is described by a transfer function

$$H(s) = \frac{a_0' + a_1' s + a_2' s^2 + \cdots + a_m' s^m}{b_0' + b_1' s + b_2' s^2 + \cdots + b_n' s^n}$$

Approximate the transfer function as

$$H_{p,q}(s) = \frac{a_0 + a_1 s + a_2 s^2 + \cdots a_p s^p}{1 + b_1 s + b_2 s^2 + \cdots b_q s^q}$$

where $p < m$, $q < n$

$H_{p,q}(s)$ is the Padé approx of $H(s)$ of type $[p/q]$

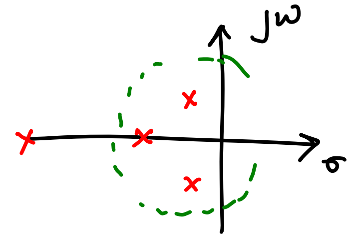
<u>Defn</u> A Padé approximant of $F(s)$ is denoted by

$$[p/q] = \frac{N_p(s)}{D_q(s)}$$

Consider $H(s)$ with distinct poles $p_1, p_2, \ldots p_n$

$$H(s) = \sum_{i=1}^{n} \frac{k_i}{s-p_i}$$

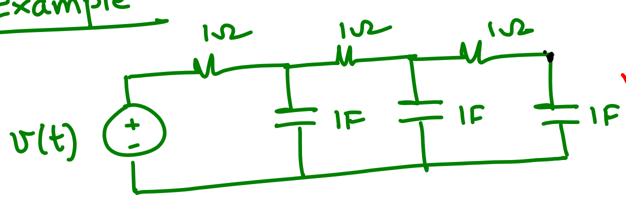
$$h(t) = \sum_{i=1}^{n} k_i e^{p_i t}$$

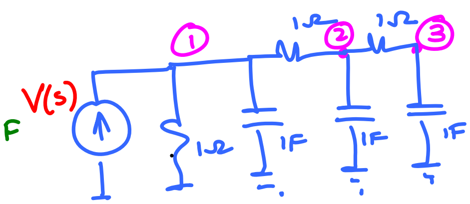


$$H_{p,q}(s) = \sum_{i=1}^{q} \frac{k_i'}{s-p_i'}$$

$$\tilde{h}(t) = \sum_{i=1}^{q} k_i' e^{p_i' t}$$

<u>Example</u>





$$\begin{bmatrix} 2+s & -1 & 0 \\ -1 & 2+s & -1 \\ 0 & -1 & 1+s \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} I(s) \\ 0 \\ 0 \end{bmatrix}$$

$$H(s) = \frac{V_3(s)}{V(s)} = \frac{1}{s^3 + 5s^2 + 6s + 1}$$

For $V(s) = \frac{1}{s}$, i.e. a step function

$$V_3(s) = \frac{1}{s(s^3 + 5s^2 + 6s + 1)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s + 3.247} + \frac{K_3}{s + 1.555} + \frac{K_4}{s + 0.1981}$$

$$v(t) = 1 - 0.0597\, e^{-3.247t} + 0.2801\, e^{-1.555t} - 1.2204\, e^{-0.1981t}$$

Padé approximations

$$[0/1] = \frac{a_0}{1+b_1 s} = \frac{1}{s^3+5s^2+6s+1}$$

$$a_0 = 1 ; \quad b_1 = 6$$

$$= \frac{1}{1+6s}$$

$$[1/2] = \frac{a_0 + a_1 s}{1+b_1 s + b_2 s^2} = \frac{1}{s^3+5s^2+6s+1}$$

$$(a_0 + a_1 s)(s^3+5s^2+6s+1) = 1 + b_1 s + b_2 s^2$$

$$a_0 = 1$$

$$a_1 + 6a_0 = b_1$$

$$+6a_1 + 5a_0 = b_2$$

$$a_0 + 5a_1 = 0$$

$$\Rightarrow \quad a_1 = -\frac{1}{5} \quad b_1 = 29/5 \quad b_2 = 19/5$$

$$H_{1,2}(s) = \frac{1 - \frac{1}{5}s}{1 + \frac{29}{5}s + \frac{19}{5}s^2}$$

Step input: $V_3(s) = \frac{K_1}{s} + \frac{K_2}{s+1.3282} + \frac{K_3}{s+0.1981}$

$$v(t) = 1 + 0.2219\, e^{-1.3282t} - 1.2219\, e^{-0.1981t}$$

A Padé approximant of $F(s)$ is

$$[p/q] = \frac{N_p(s)}{D_q(s)}$$

Padé table

q \ p	0	1	2	. . . .
0	[0/0]	[1/0]	[2/0]	
1	[0/1]	[1/1]	[2/1]	
2	[0/2]	[1/2]	[2/2]	
⋮				

AWE McLaurin series expansion

$$F(s) = m_0 + m_1 s + m_2 s^2 + \cdots = \sum_{i=0}^{\infty} m_i s^i$$

m_i is called the ith moment

Recall $F(s) = \int_0^\infty f(t) e^{-st} dt$ Laplace transform

$$= \int_0^\infty f(t) \left[1 - st + \frac{(st)^2}{2!} - \frac{(st)^3}{3!} + \cdots \right] dt$$

$$= \int_0^\infty f(t) dt - s \int_0^\infty t f(t) dt + \frac{s^2}{2!} \int_0^\infty t^2 f(t) dt$$

$m_0 = \int_0^\infty f(t) dt$; $m_i = \frac{(-1)^i}{i!} \int_0^\infty t^i f(t) dt$

Suppose $m_0, m_1, m_2, \ldots$ are known. Then
Consider the $[q-1/q]$ Padé approximant

$$m_0 + m_1 s + m_2 s^2 + m_3 s^3 + \cdots + m_{2q-1} s^{2q-1} = \frac{a_0 + a_1 s + a_2 s^2 + \cdots a_{q-1} s^{q-1}}{1 + b_1 s + b_2 s^2 + \cdots b_q s^q}$$

s^0: $a_0 = m_0$

s^1: $a_1 = m_1 + m_0 b_1$

s^2: $a_2 = m_2 + m_1 b_1 + m_0 b_2$

⋮

s^{q-1}

$s^q \qquad 0 = m_q + m_{q-1} b_1 + m_{q-2} b_2 + \cdots + m_0 b_q$

$s^{2q-1} \qquad 0 =$

$$\begin{bmatrix} m_0 & m_1 & \cdots & m_{q-1} \\ m_1 & m_2 & \cdots & m_q \\ m_2 & m_3 & \cdots & m_{q+1} \\ \vdots & & & \\ m_{q-1} & m_q & \cdots & m_{2q-1} \end{bmatrix} \begin{bmatrix} b_q \\ b_{q-1} \\ \vdots \\ b_1 \end{bmatrix} = \begin{bmatrix} m_q \\ m_{q+1} \\ \vdots \\ m_{2q-1} \end{bmatrix}$$

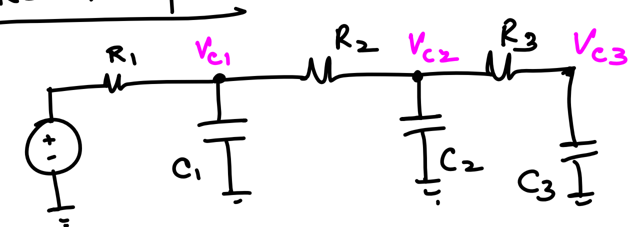
How do we compute the moments?

These are moments of the impulse response

$v(t) = \delta(t) \Rightarrow V(s) = 1$

Consider the RC example

$V(s) = 1$



Expressing V_{c1}, V_{c2}, V_{c3} as an infinite power series

$$V_{c1} = m_0^{c_1} + m_1^{c_1} s + m_2^{c_1} s^2 + \cdots$$

$$V_{c2} = m_0^{c_2} + m_1^{c_2} s + m_2^{c_2} s^2 + \cdots$$

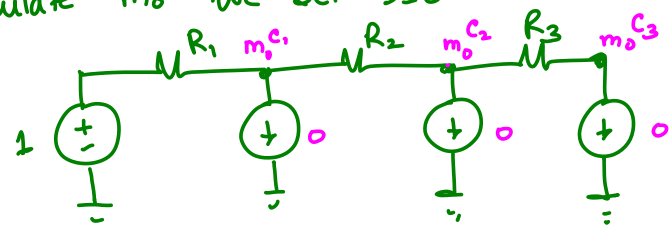
$$V_{c3} = m_0^{c_3} + m_1^{c_3} s + m_2^{c_3} s^2 + \cdots$$

Current through capacitor C_i

$$I^{c_i} = sC_i V_{c_i}$$

$$= sC_i \left[m_0^{c_i} + m_1^{c_i} s + m_2^{c_i} s^2 + \cdots \right]$$

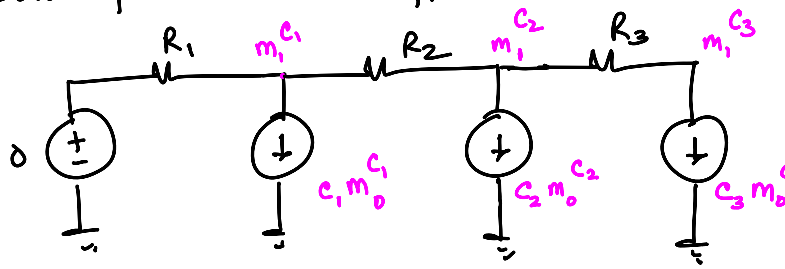
To calculate m_0 we set $s = 0$



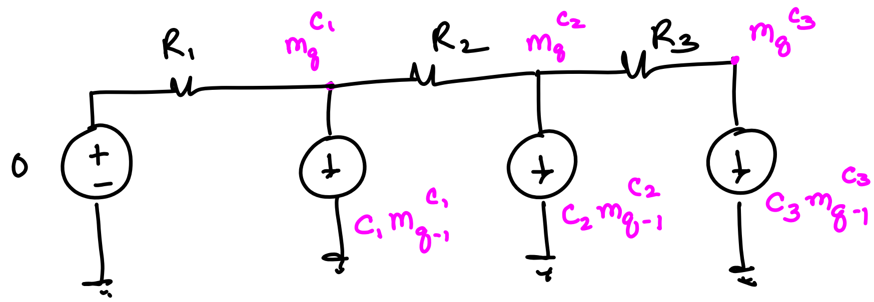
$\Rightarrow m_0^{c_1}, m_0^{c_2}, m_0^{c_3}$ are known

$= 1$

Now solve for the s^1 coefficients



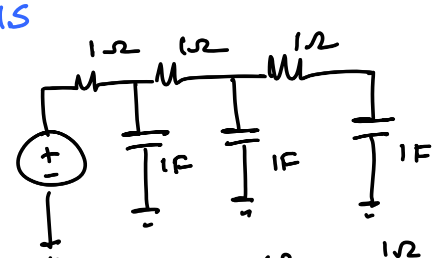
$\vdots$



- We are obtaining the dc solution of a network with capacitors replaced by indep current sources (inductors → indep voltage srcs)

- For each moment calculation the circuit topology remains the same. Only the RHS changes
  - ⇒ LU factor circuit matrix & solve using forward/back solves for different RHS

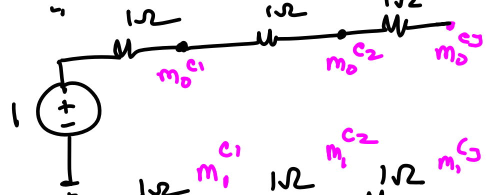
RC Example



$m_0(1) = 1$

$m_0(2) = 1$

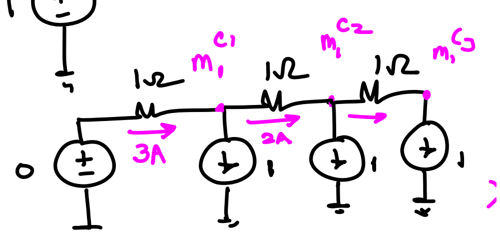
$m_0(3) = 1$



$m_1(1) = -3$

$m_1(2) = -5$

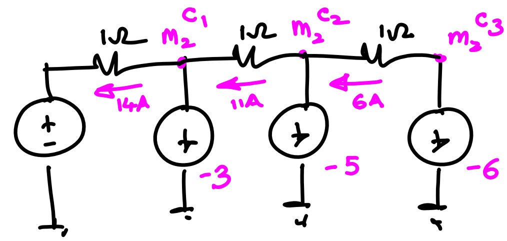
$m_1(3) = -6$



$m_2(1) = 14$

$m_2(2) = 25$

$m_2(3) = 31$



$$V_3(s) = 1 - 6s + 31s^2 - 157s^3$$

$$[1/2]: \quad = \frac{a_0 + a_1 s}{1 + b_1 s + b_2 s^2}$$

$$(1 - 6s + 31s^2 - 157s^3)(1 + b_1 s + b_2 s^2) = a_0 + a_1 s$$

$$\left.\begin{aligned} a_0 &= 1 \\ a_1 &= b_1 - 6 \\ 0 &= b_2 - 6b_1 + 31 \\ 0 &= \ldots \end{aligned}\right\} \quad \begin{aligned} b_1 &= 29/5 \\ b_2 &= 19/5 \\ a_1 &= -1/5 \end{aligned}$$