ECE 521 Lecture # 18 Nov. 23, 2016

Note Title 11/23/2016

Graded Exams handed back

Hi 93/100 Lo 72/100

Avg 82.7/100 (Solutions posted)

Project presentations Wed Nov. 30

- Guidelines emailed

FINAL PROJECT REPORTS Due Thu DEC 8 by noon

All Scores ARE posted on Canvas

- please report any discrepancy

HW - part 1: ✓ = 10; ✓- = 8; ✓-- = 7

No CLASS MONDAY Nov. 28

Use time to work on project/presentation

Today - wrap up RF simulation

Simulation of Radio Frequency Integrated Circuits

Based on:

- **K. Mayaram, D. C. Lee, S. Moinian, D. A. Rich, and J. Roychowdhury, "Computer-Aided Circuit Analysis Tools for RFIC Simulation: Algorithms, Features, and Limitations," IEEE Trans. CAS-II, April 2000.**
- **Slides from Yutao Hu, Volodymr Kratyuk, Xiaochun Duan, and Igor Vytyaz**

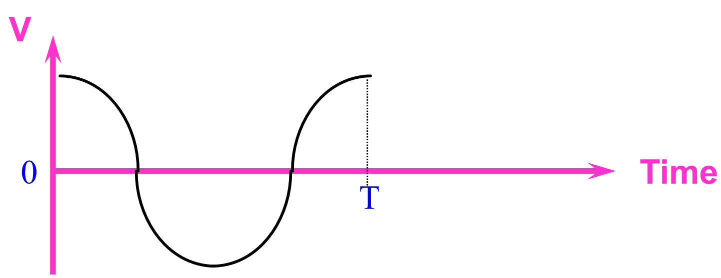
Outline

- **Time-domain Periodic Steady State Analysis (PSS)**
  - Shooting method
  - Oscillator simulation
- **Frequency-domain PSS**
  - Harmonic balance method
    - **Single tone, two tone**
  - Oscillator simulation
- **Noise in nonlinear circuits**
  - Oscillator phase noise analysis

Time-Domain Method

- **Impose periodicity constraint**

  v(0) = v(T)



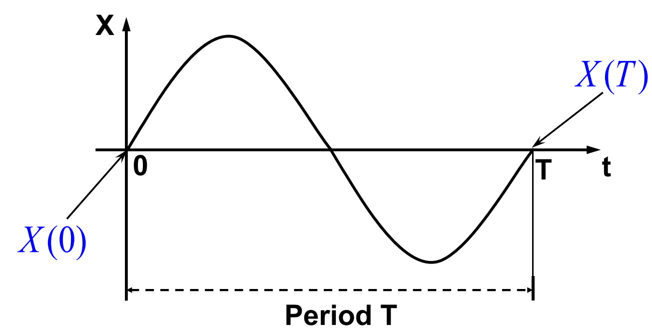
- **For a driven circuit period T is known**
- **For an oscillator T is an unknown**

Time-Domain Shooting Method

- **Periodicity constraint**

  ⇒ **Two-point boundary value problem:**

$$X(0) - X(T, X(0)) = 0$$



Time-Domain Shooting Method

- **Solve using Newton's algorithm**

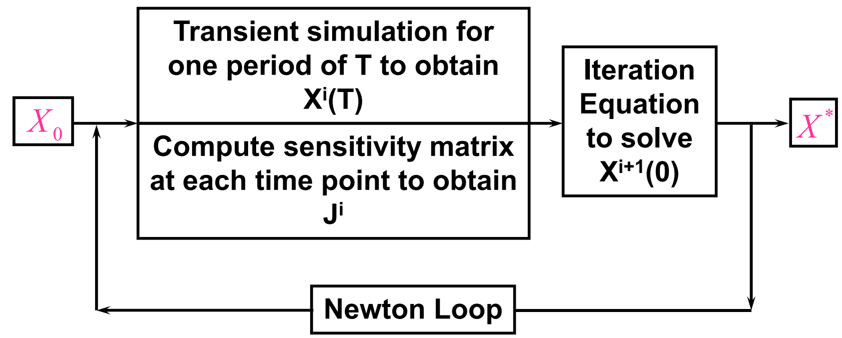
$$X^{i+1}(0) = X^i(0) - [I - J^i]^{-1}[X^i(0) - X^i(T)]$$

where $J^i = \dfrac{\partial X^i(T, X(0))}{\partial X^i(0)}$ **when T is fixed**

I **is the identity matrix**

– Final state $X^i(T)$ is obtained by one period transient
– Sensitivity matrix J has initial value I and is computed in the one period transient
– For autonomous systems, period T is an unknown

Solution Procedure



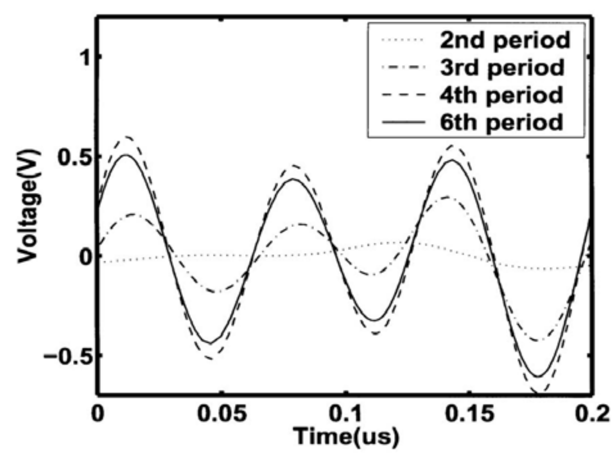
Modifications and heuristics for efficiency and convergence reliability

Implementation Considerations

- **Heuristics for autonomous systems (oscillators)**
  – Period (T) is an unknown
  – Three period transient without Newton's iteration in the beginning to eliminate fast transients in the circuit
  – Sensitivity computed when error below a threshold
  – Damped Newton's iteration
  – ΔT is limited to 10% of current period

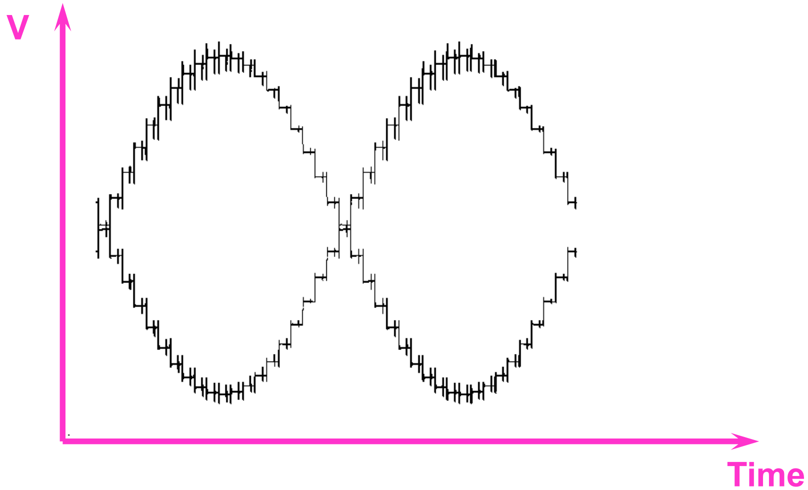
Examples and Results

- **Frequency multiplier**
  – Shooting method: 6 periods
  – Conventional transient: 1500 periods



Example

- **Switched capacitor 5th-order elliptic filter**
  - 245 MOSFETs, 171 nodes



Time-domain steady-state method is efficient

Example circuits	Conventional transient simulation (# of periods)	Time-domain steady-state method (# of periods)
DC supply	80	6
CB amplifier	30	4
EC xfrmr osc.	185	25
Freq. Multiplier	1500	6
LC EC osc	22	9
SCP amplifier	182	6
H.F. Colpitts	20	12
L.F. Colpitts	84	18
Demodulator	12000	4

Oscillator Periodic Steady-state analysis

- **Problem formulation**

$$x(0) = x(T)$$

or

$$x(0) = \Phi(x(0),0,T)$$

- **In autonomous systems (oscillators):**
  - T is an unknown
  - Hence, n equations in n+1 unknowns

Solution for oscillators

- **Add an extra equation:**

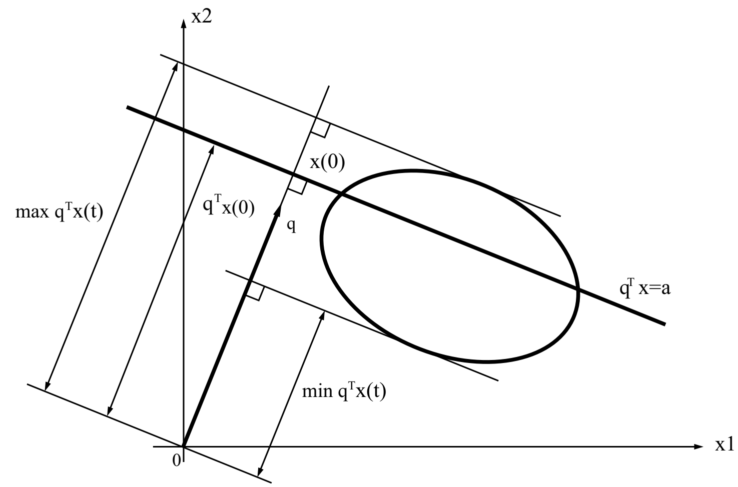
$$\begin{cases} x(0) = \Phi(x(0),0,T) \\ q^T x(0) = a \end{cases}$$

Solution for oscillators

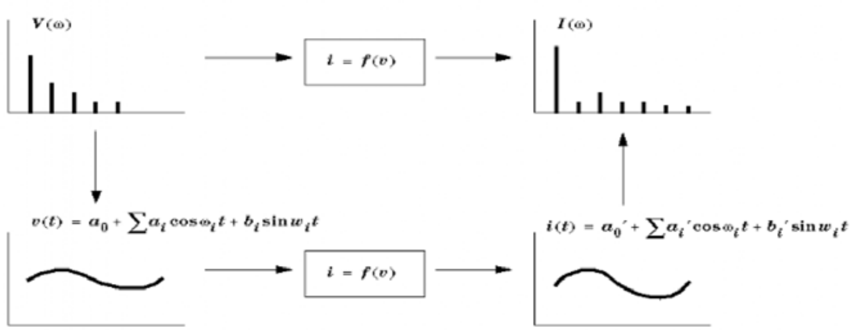
- Impose constraints for the added equation:

$$\begin{cases} \max_t q^T x(t) > a \\ \min_t q^T x(t) < a \\ q^T \dfrac{dx(0)}{dt} \neq 0 \end{cases}$$

Graphical solution for oscillators

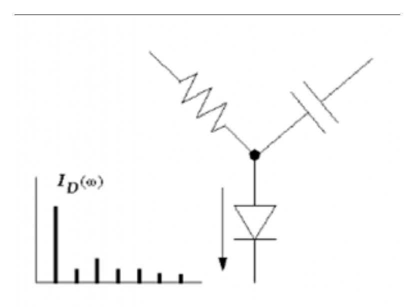


Non-Linear Frequency Domain Analysis



- Low distortion signals require few Fourier series coefficients
- Smooth device models are essential for RF

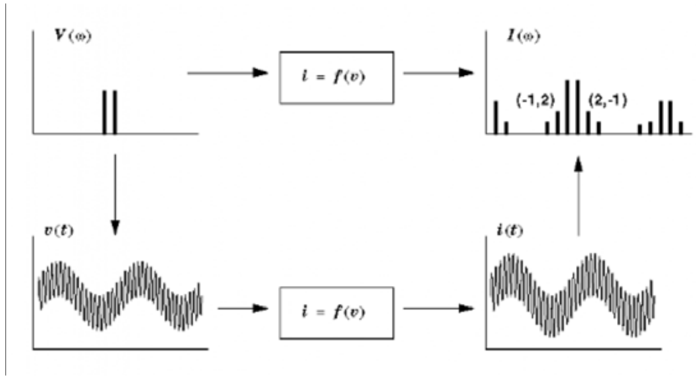
Harmonic Balance



- "Balance" the frequency spectrum at each node

- Time-derivatives (capacitors) become multiplication in frequency domain
- Handle distributed elements in freq. domain

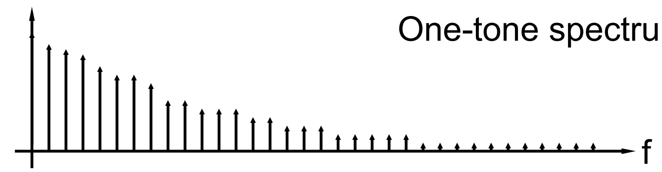
Multi-Tone Frequency Domain Analysis

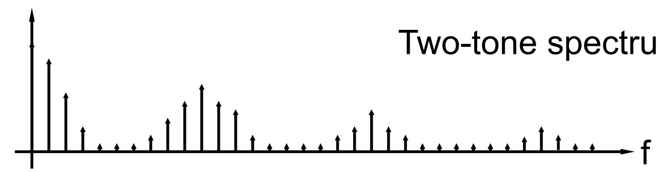


- **Minimum number of “time-domain” samples dictated by the number of significant Fourier coefficients, not by the Nyquist rate**

Frequency Truncation

- Harmonic truncation
  - keep a finite number of frequencies containing significant energy





Harmonic Balance: Summary

- **Conventional HB simulators**
  - Small circuits
  - Large memory requirements
  - Not well suited for RFIC simulation
- **State-of-the Art** **(Univ. of Bremen, Bell Labs)**
  - Circuits with tens to hundreds of transistors
  - Sophisticated IC device models
  - Run time and memory required almost linear with size of circuit and number of Fourier coefficients

Harmonic Balance Method

- **Truncated Fourier series approximation of x(t)**

$$x(t) \approx a_0 + \sum_{i=1}^{s} \left(a_i \cos(\omega_i t) + b_i \sin(\omega_i t)\right)$$

- **For 2s+1 time samples $x_0 \ldots x_{2s}$**

$$x = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{2s} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & \cos(\omega_1 t_0) & \cdots & \sin(\omega_s t_0) \\ 1 & \cos(\omega_1 t_1) & \cdots & \sin(\omega_s t_1) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \cos(\omega_1 t_{2s}) & \cdots & \sin(\omega_s t_{2s}) \end{bmatrix}}_{\Gamma^{-1}} \underbrace{\begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ b_s \end{bmatrix}}_{X} = \Gamma^{-1} X$$

Harmonic Balance Method

- **System equation in time domain:**

$$i(x(t), t) + \frac{d}{dt} q(x(t), t) + s(t) = 0$$

  - x(t) the vector of circuit waveforms
  - i is a vector of contributions from nonreactive elements
  - q is a vector of contributions from reactive elements
  - s stimulus vector

Harmonic Balance Method

- **Frequency-domain representation**

$$\Gamma i(\Gamma^{-1}X) + \Omega \Gamma q(\Gamma^{-1}X) + S = 0$$

Where Ω is representation of derivative operation

$$\Omega = \begin{bmatrix} 0 & & & \\ & \varpi_1 & & \\ & & \ddots & \\ & & & \varpi_s \end{bmatrix}, \quad \varpi_i = \begin{bmatrix} 0 & \omega_i \\ -\omega_i & 0 \end{bmatrix}$$

- Jacobian matrix in Newton iteration

$$\Gamma G \Gamma^{-1} X + \Omega \Gamma C \Gamma^{-1} X$$

Two Tone Harmonic Balance Method

- Frequency-remapping for multi-tone analysis
  - To use standard DFT
  - Fourier coefficients of function independent of the actual values of frequencies

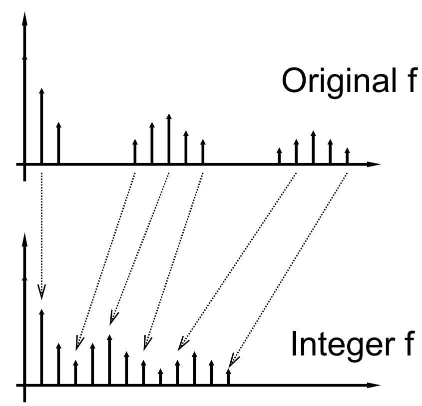
$$\Gamma i(\Gamma^{-1}V) = \hat{\Gamma} i(\hat{\Gamma}^{-1}V) \qquad \Gamma q(\Gamma^{-1}V) = \hat{\Gamma} q(\hat{\Gamma}^{-1}V)$$

$$(\omega_0 \cdots \omega_s) \xrightarrow{\text{remapping}} (\hat{\omega}_0 \cdots \hat{\omega}_s)$$

$$\hat{\Gamma} i(\hat{\Gamma}^{-1}X) + \hat{\Omega} \hat{\Gamma} q(\hat{\Gamma}^{-1}X) + S = 0$$

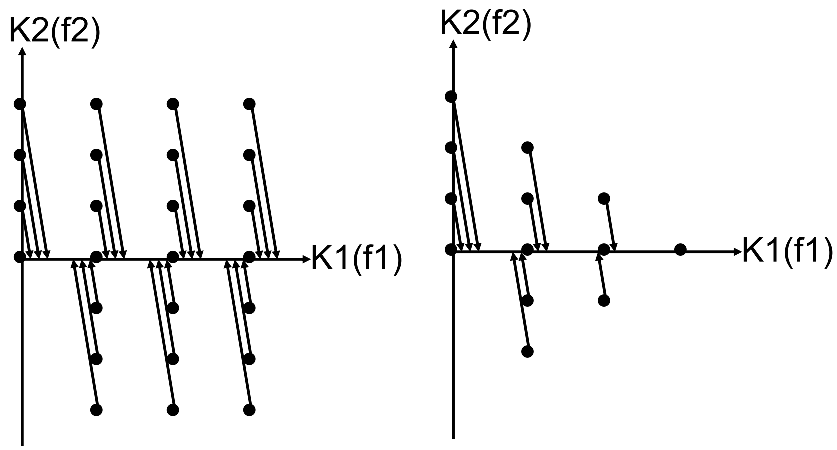
Two Tone Harmonic Balance Method

- Frequency-remapping to integer frequencies



Integer frequency	Original frequency
0	0*f1+0*f2
1	0*f1+1*f2
2	0*f1+2*f2
3	1*f1-2*f2
4	1*f1-1*f2
5	1*f1+0*f2
6	1*f1+1*f2
7	1*f1+2*f2
8	2*f1-2*f2
9	2*f1-1*f2
10	2*f1+0*f2
11	2*f1+1*f2
12	2*f1+2*f2

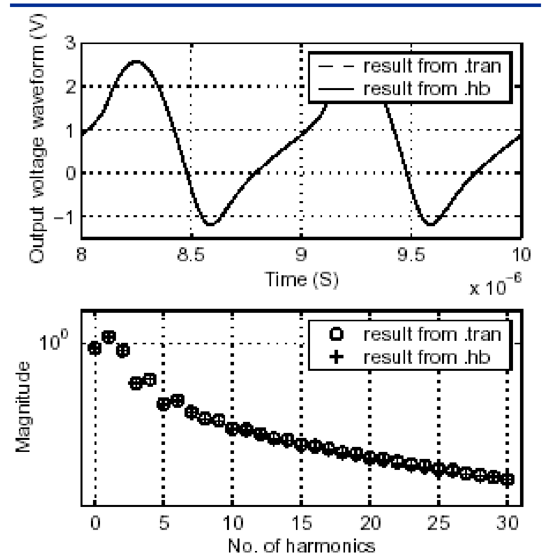
Two-Tone Harmonic Truncation Methods



(a) box truncation (b) diamond truncation

Examples and Results

- **Simple rectifier**



– 30 harmonics
– No. of iterations=16
– Result verified by transient simulation

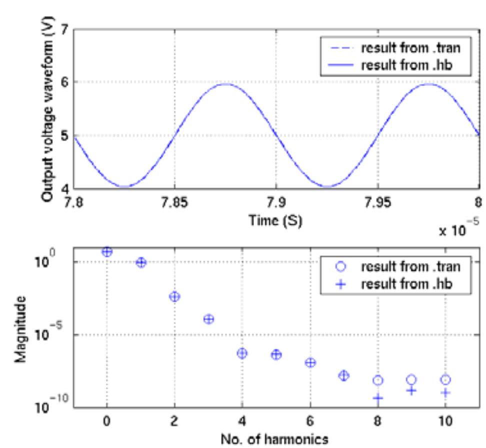
Examples and Results

- A single-BJT mixer circuit
  - 27 frequencies chosen by box truncation
  - Harmonic balance method takes 11 iterations
  - Transient analysis is impractical

f_{LO}=500MHz, f_{RF}=499.9MHz
output filter: Q=100, f_{center}=100KHz
Fundamental of this mixer: 100KHz
The fifth harmonic of LO: 2500MHz
⇒ 50,000 time points per cycle
X many cycles needed by high Q output filter

Examples and Results

- **MOSFET common source tuned amplifier**



– 10 harmonics
– No. of iterations=6
– Result verified by transient simulation

Oscillator Simulation with HB

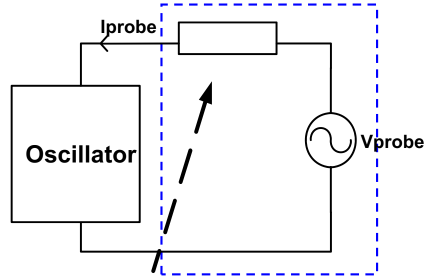
- **Problems**
  - Unknown period of oscillation
  - Arbitrary time origin
- **Solutions** (K. Kundert, 1990)
  - Frequency as an additional unknown
  - Additional equation to fix phase

$$\mathbf{F}(\mathbf{X},\omega) = \Gamma \mathbf{i}(\Gamma^{-1}\mathbf{X}) + \Omega(\omega)\Gamma \mathbf{q}(\Gamma^{-1}\mathbf{X}) + \mathbf{S} = 0$$

$$\mathbf{X}_m^s(1) = 0$$

  - Direct implementation ⇒ convergence problems

Use Voltage Probe



$$Z(\omega) = \begin{cases} 0, & \omega = \omega_f \\ \infty, & \omega \neq \omega_f \end{cases}$$

- Convergence criterion
  - Probe current equals zero
- Advantages
  - Autonomous circuit ⇒ forced circuit

E.Ngoya, *Int.J. Microw. Milim.-wave CAE,*1995

Two-Level Newton Method

- **Bottom Level:** Voltage Probe Forced Circuit

$$\begin{bmatrix} \mathbf{J} & \mathbf{e}_m^c(1) & \mathbf{e}_m^s(1) \\ \mathbf{e}_m^c(1)^T & 0 & 0 \\ \mathbf{e}_m^s(1)^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{X}^{(j+1)} \\ \mathbf{I}_{probe}^{c(j+1)} \\ \mathbf{I}_{probe}^{s(j+1)} \end{bmatrix} = \begin{bmatrix} \mathbf{RhsF} \\ \mathbf{V}_{probe} \\ 0 \end{bmatrix}$$

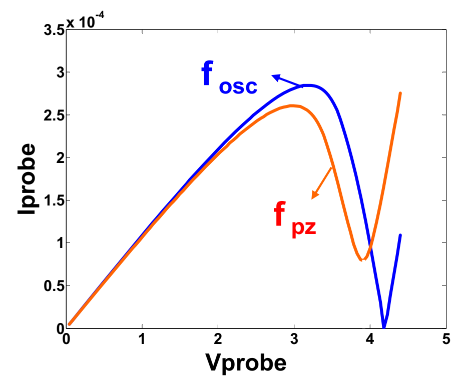
- **Top Level:** Probe Equation

$$\mathbf{I}_{probe}^{c}(\mathbf{V}_{probe},\omega) = 0$$

$$\mathbf{I}_{probe}^{s}(\mathbf{V}_{probe},\omega) = 0$$

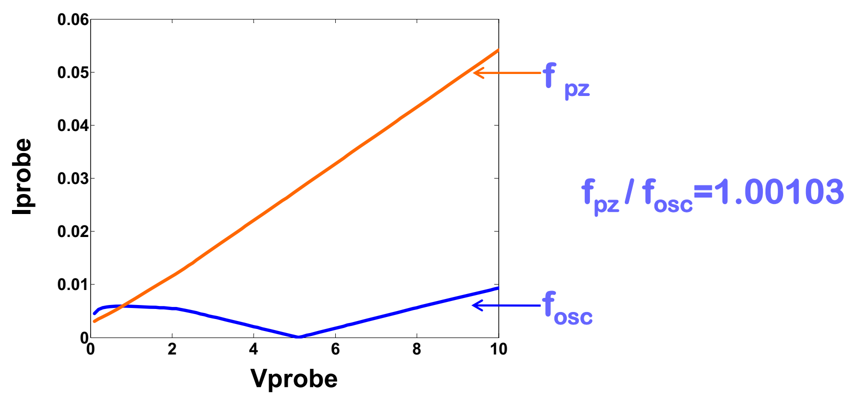
Initial Condition – Sinusoidal Oscillators

- **Initial guess for oscillation frequency and probe voltage**
  - Pole-zero analysis for initial frequency
  - Probe voltage swept at a fixed frequency for initial probe voltage



f_{pz}: frequency from pole-zero analysis
f_{osc}: actual oscillation frequency

Difficulty with High Q Oscillator



- **Problem:** Hard to get initial guess for probe voltage
- **Solution:** Method requiring no initial guess

Homotopy Methods

- Original Problem

  $F(X)=0 \qquad F: R^n \to R^n$

- Homotopy

  $H(X, \lambda)=0 \qquad H: R^n \times R \to R^n$

  $\lambda=0$: $H(X,0)=G(X)$ ⟶ Easy solution

  $\lambda=1$: $H(X,1)=F(X)$ ⟶ Original problem

- Key for convergence
  - Properly embed λ

Homotopy Formulation for Oscillators

- Original equation

$$I^c_{probe}(V_{probe}, \omega)=0 \qquad \qquad \frac{I^c_{probe}(V_{probe}, \omega)}{V_{probe}}=0$$

$$\Rightarrow$$

$$I^s_{probe}(V_{probe}, \omega)=0 \qquad \qquad \frac{I^s_{probe}(V_{probe}, \omega)}{V_{probe}}=0$$

- Homotopy map

$$I^c_{probe}(V_{probe}, \omega, \lambda)=\lambda\frac{I^c_{probe}(V_{probe}, \omega)}{V_{probe}}+(1-\lambda)g_1(V_{probe}-V_{init})$$

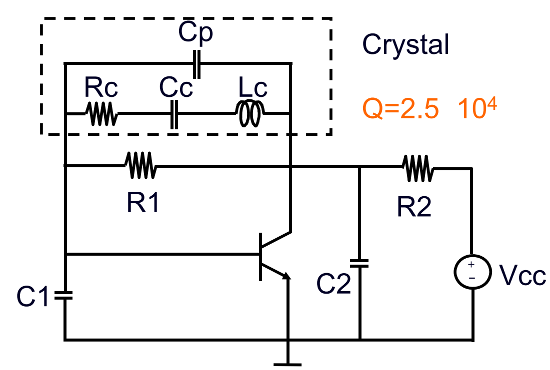
$$I^s_{probe}(V_{probe}, \omega, \lambda)=\lambda\frac{I^s_{probe}(V_{probe}, \omega)}{V_{probe}}+(1-\lambda)g_2(\omega-\omega_{init})$$

V_{init} (ω_{init}) : probe voltage (frequency) initial value

g1,g2 : scaling factors

Example: Pierce Oscillator

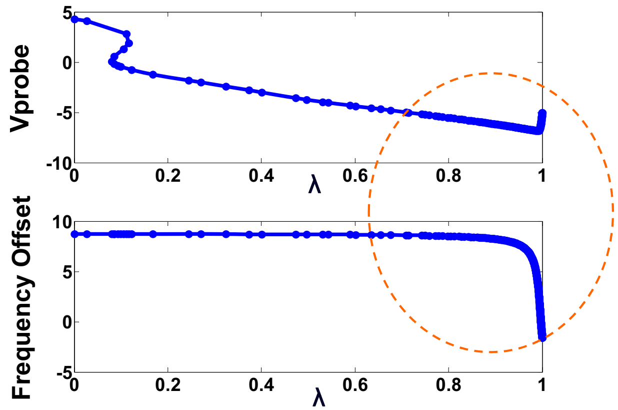
Two-level method fails



- Voltage probe at BJT collector node
  - V_{init} : DC value
  - f_{init} : f_{pz}

M.Gourary et al, *Comput. Methods Appl. Mech. Engr,* 2000

Solution Traces



Small λ increments

- Robust: Tracking turning point properly
- Slow: Large number of λ steps (>1000)

High-Q Oscillator Examples

Two-level method fails for these circuits

Circuit	Q	f_o (MHz)	f_{pz}/f_o	# Steps	#Iter	CPU time (sec)
Colpitts	1.00×10^3	1.5915	0.99987	11	208	17.5
TNT	1.02×10^3	11.795	0.84738	15	492	26.3
Pierce (BJT)	1.07×10^4	4.0811	0.97036	10	241	13.2
Pierce1 (MOS)	3.69×10^4	7.9992	0.99907	5	98	8.2
Clapp	1.14×10^5	20.124	0.99998	12	243	19.2
Pierce2 (MOS)	2.82×10^5	1.1256	0.99995	26	403	141

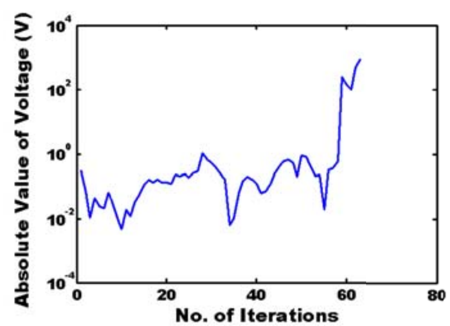
Comparison of Homotopy Method with Two-level Method

Circuit	# Iterations		CPU time (sec)	
	Two-level	Homotopy	Two-level	Homotopy
Colpitts (BJT)	63	197	1.6	5.2
TNT	243	310	12.6	23.4
Wien	93	274	6.1	14.5
Sony	88	170	5.6	12.1
Phase shift	57	307	23.9	127.2
Source coupled	74	358	12.8	60.1
Cross coupled	69	398	3.1	20.8
Colpitts (MOS) #	x	172	x	854.4
Sony #	x	593	x	581.2

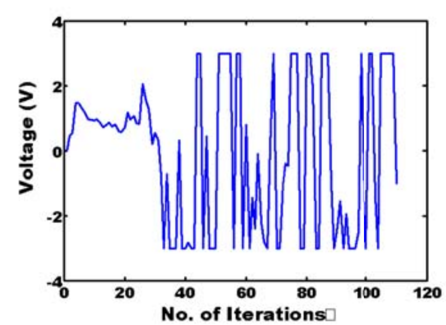
#: circuit with numerical models, x: no convergence

Difficulty with Ring Oscillator

Bottom-level circuit waveform during the Newton iterative process



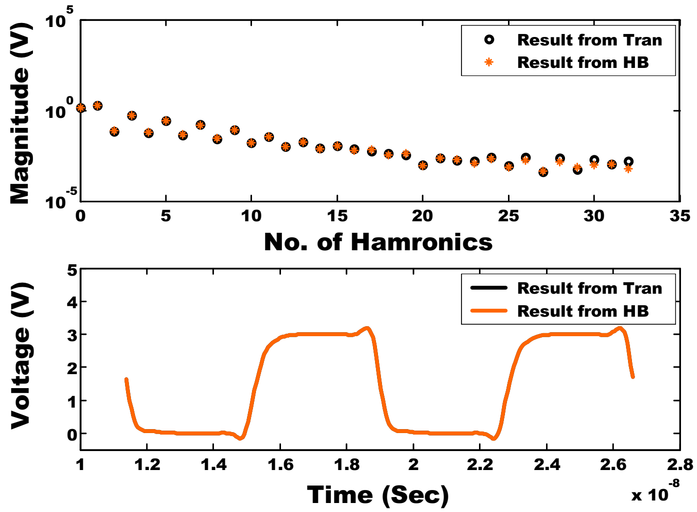
Divergence with a small damping factor



Oscillation with voltage/current thresholds

Examples and Results

- 9 stage single-ended ring oscillator

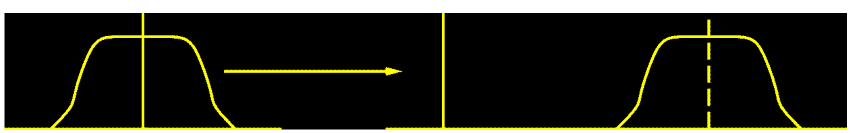


Robust Harmonic Balance Method for Oscillators

- Homotopy-based harmonic balance method for high-Q oscillator simulation
- Single delay cell equivalent circuit for ring oscillator simulation with identical delay cells
- Multiple-probe method for general ring oscillator simulation

Mixing Noise

- Up/down conversion of noise due to mixing

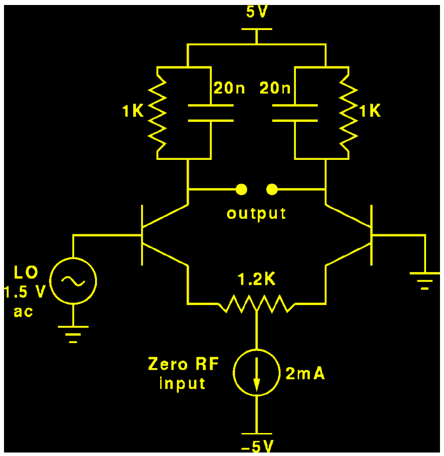


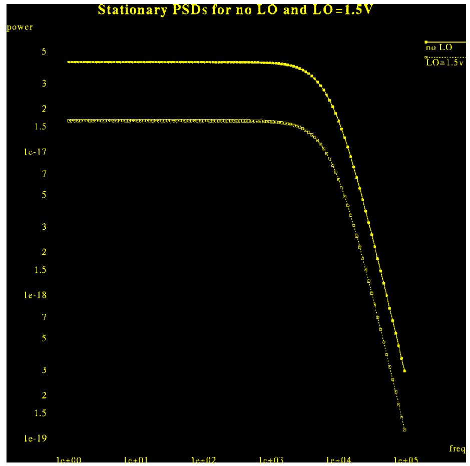
- SPICE noise analysis does not work
- Cyclostationarity/frequency correlation important
- Monte Carlo or stochastic methods

Noise Analysis Methods

- Monte Carlo Methods
  - OK for arbitrarily large noise
  - Inaccurate for small noise
  - Time consuming
- Stochastic Methods
  - Mainly used for small noise
  - Can be very efficient and accurate

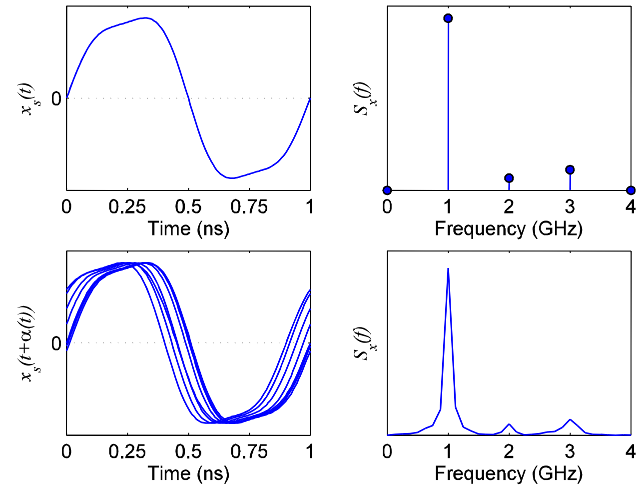
Noise in a Switching Mixer





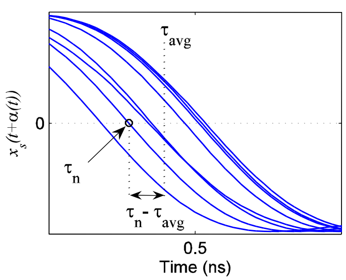
Ideal and Noisy Oscillators

- **Ideal**
- **Noisy**



– Noise characterization
  - **Timing jitter**
  - **Phase noise**

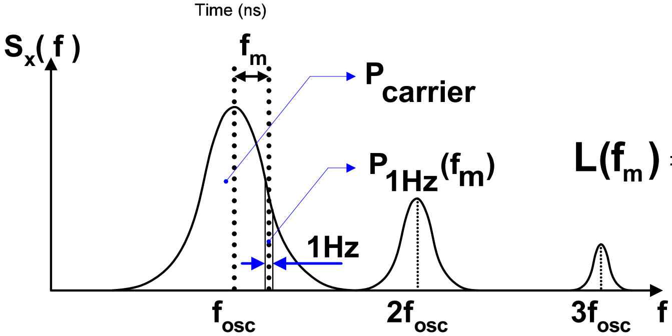
Timing Jitter and Phase Noise



- **Timing jitter**

$$\sigma_c^2 = \lim_{N\to\infty}\left(\frac{1}{N}\sum_{n=1}^{N}\left(\tau_n - \tau_{avg}\right)^2\right)$$

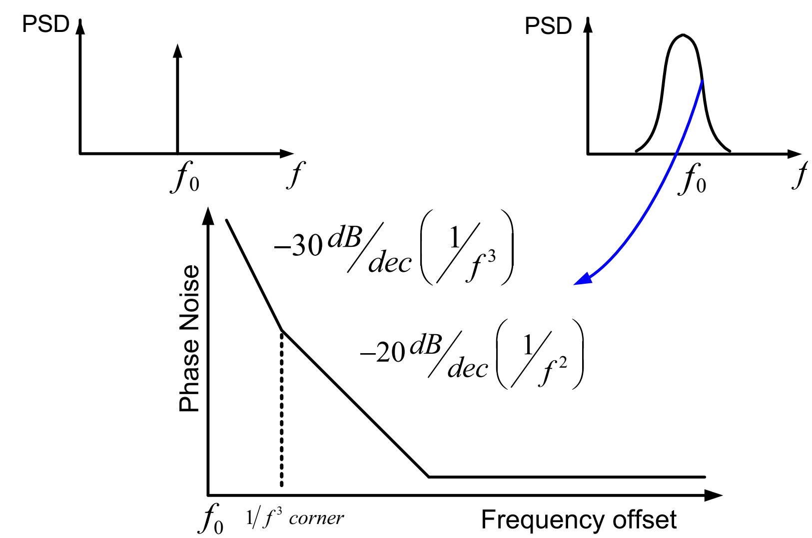
- **Phase noise**



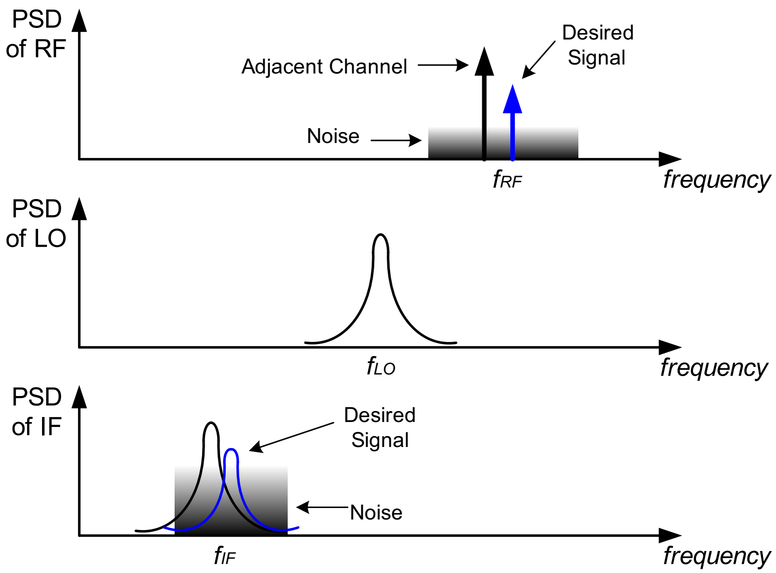
$$L(f_m) = 10\log_{10}\frac{P_{1Hz}(f_m)}{P_{carrier}}$$

Power Spectral Density (PSD) of Oscillators

Ideal Oscillator **Real Oscillator**

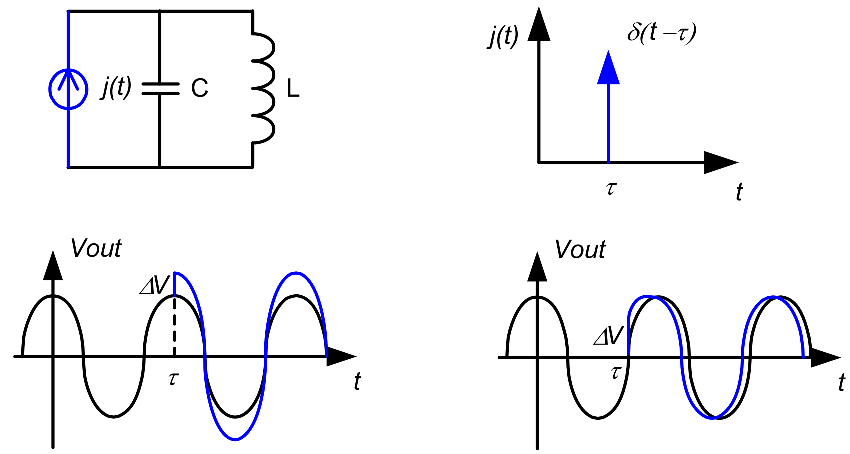


The Effect of the Local Oscillator Phase Noise



Noisy oscillators

- Noise can perturb both the amplitude and phase of an oscillator



Phase Noise

- **Important for adjacent channel interference, data recovery, and sampled data systems**
- **Mixing noise methods can handle phase noise away from carrier**
  - Inaccurate close to carrier
- **Most analyses are of specific oscillators under simplifying assumptions**
- **New methods for proper phase noise calculation available in Spectre-RF, Eldo-RF**

Methods for Phase noise Calculation

Hajimiri & Lee Theory:

- Only transient analysis is needed for the simulations
- Transient analysis is needed for each node perturbed by noise
- Impulse sensitivity function (ISF) has to be extracted using post processing

Demir's Theory:

- Require only steady-state solution
- Allows for efficient simulation of large circuits
- The contribution of each noise source can be obtained easily
- Difficult to implement if a steady-state analysis is not available

Phase Noise Equations

- A system of DAEs for a circuit with noise sources

$$\frac{d}{dt}q(x)+g(x)+B_w(x)b_w(t)+\sum_{m=1}^{M}B_{cm}(x)b_{cm}(t)=0$$

circuit currents | currents from white noise sources | currents from colored noise sources

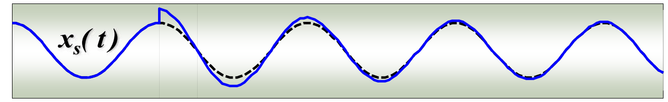
- Single-sideband phase noise spectrum in dBc/Hz

$$L(f_m)=10\log_{10}\left(\frac{f_o^2 c(f)}{f_m^2}\right)$$

A. Demir et al., "Phase noise in oscillators: a unifying theory and numerical methods for characterization," *IEEE Trans. CAS-I*, May 2000

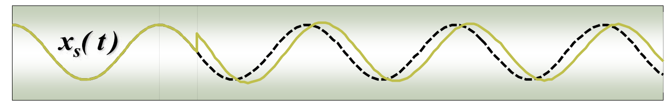
Perturbation projection vector (PPV)

- **Effect of a perturbation on oscillator**
  - Phase deviation is zero



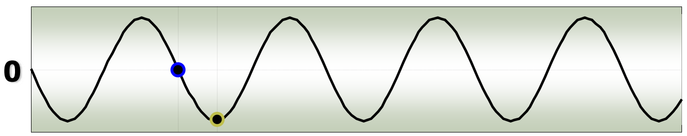
$\Delta\varphi = 0$

  - Phase deviation persists



$\Delta\varphi < 0$

- **Perturbation projection vector (PPV)**

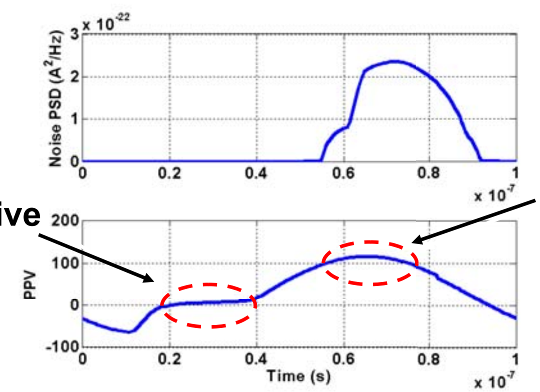


$v_1(t) \propto \Delta\varphi$

- **PPV facilitates accurate phase noise computation**

Perturbation Projection Vector (PPV)

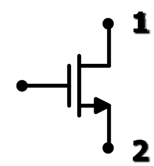
- **Periodic vector**
  - Transfer function from noise sources to phase noise
  - Similar to Hajimiri and Lee's impulse sensitivity function (ISF)



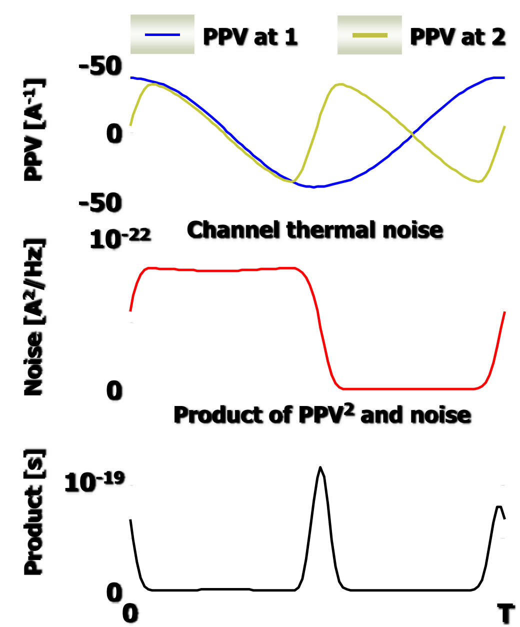
Less sensitive to noise

More sensitive to noise

PPV in the time domain

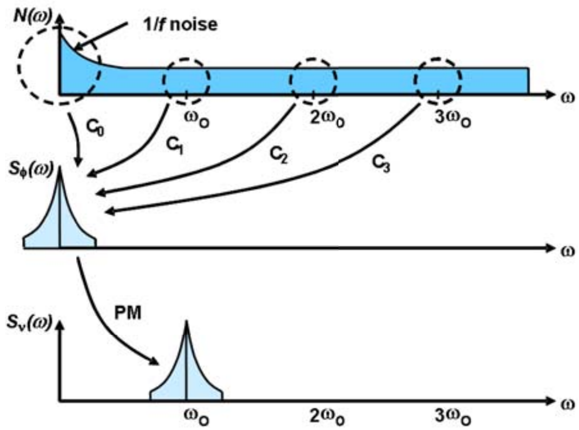


- **PPV is a transfer function from noise source to phase**
- **Phase noise is area under PPV^2 $\times$ noise curve**

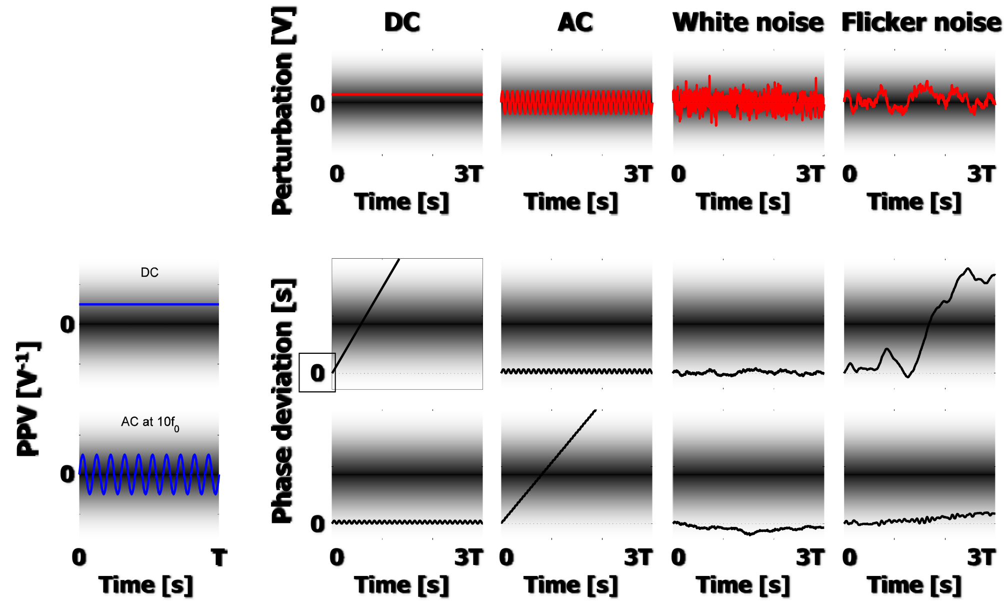


PPV in the frequency domain

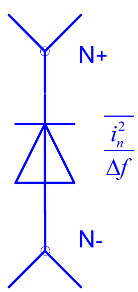
- **Flicker noise up-conversion**
  - due to DC term of PPV
- **High frequency noise gets folded**
  - due to harmonics of PPV



Practical implication of PPV



Mapping Matrix for White noise sources



Double-Sideband PPSD

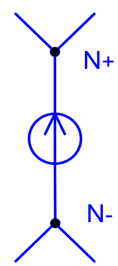
Stationary white noise process with PSD=1

$$\frac{\overline{i_n^2}}{\Delta f} = qI_{DC}\left(x(t)\right) \Rightarrow b_w(t) = \begin{bmatrix} \cdots \\ u(t) \\ \cdots \\ \cdots \\ \cdots \\ \cdots \end{bmatrix}_{p \times 1}$$

$$B_w\left(x(t)\right) = \begin{bmatrix} \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & +\sqrt{qI_{DC}\left(x(t)\right)} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & -\sqrt{qI_{DC}\left(x(t)\right)} & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}_{n \times p} \begin{matrix} \\ \text{N+} \\ \\ \text{N-} \\ \\ \end{matrix}$$

Mapping Vector for Colored Noise Sources

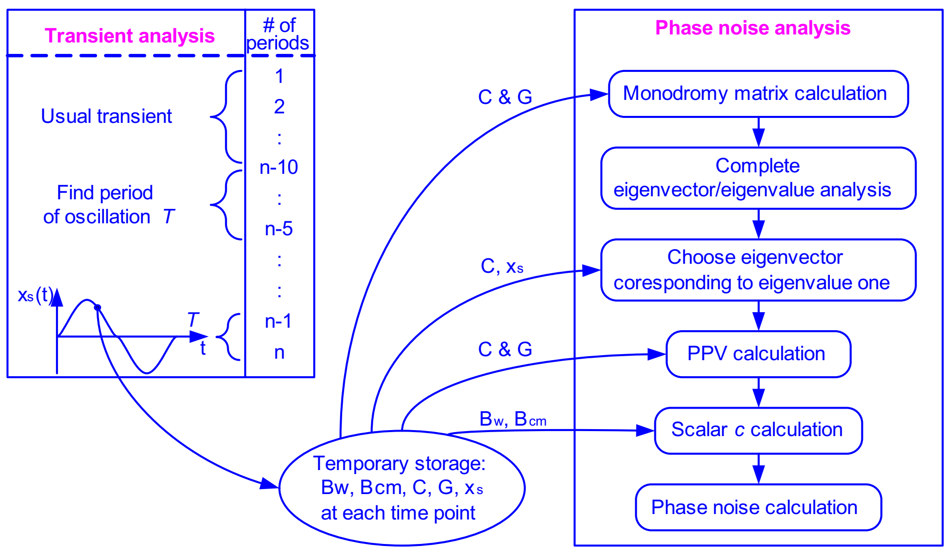
- Mapping vector for colored noise sources has to be calculated for each frequency of interest



Double-Sideband PSD

$$\frac{\overline{i_n^2}}{\Delta f} = \frac{K_f I_{ds}^{af}\left(x(t)\right)}{2C_{ox}L_{eff}^2 f^{ef}} \Rightarrow B_{cm}\left(x(t)\right) = \begin{bmatrix} \cdots \\ +\sqrt{\frac{K_f I_{ds}^{af}\left(x(t)\right)}{2C_{ox}L_{eff}^2 f^{ef}}} \\ \cdots \\ -\sqrt{\frac{K_f I_{ds}^{af}\left(x(t)\right)}{2C_{ox}L_{eff}^2 f^{ef}}} \\ \cdots \end{bmatrix}_{n \times 1} \begin{matrix} \\ \text{N+} \\ \\ \text{N-} \\ \\ \end{matrix}$$

Phase Noise Analysis Implementation



Measured RMS jitter of 1.4 GHz PLL

- **PPV-based prediction agrees with measurement**