ECE 521 Fall 2010

Test (11/22/10)

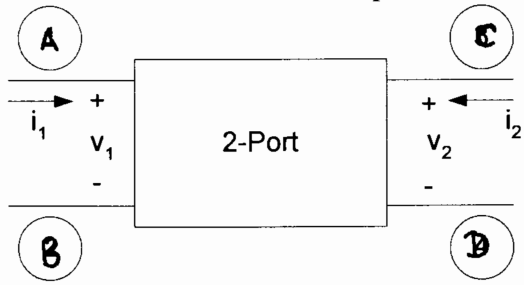
Total # Pages 5
Total # Problems 4

Name SOLUTION

1.	(20 points)	________
2.	(35 points)	________
3.	(10 points)	________
4.	(35 points)	________
Total	(100 points)	________

GOOD LUCK

1. The 2-port shown below is defined in terms of its s-parameters.



The s-parameters are defined as

$$b_1 = s_{11}a_1 + s_{12}a_2$$
$$b_2 = s_{21}a_1 + s_{22}a_2$$

a_k and b_k are related to v_k and i_k for $k=1,2$ by

$$a_k = v_k + z_0 i_k$$
$$b_k = v_k - z_0 i_k$$

where z_0 is the characteristic impedance. Write the MNA stamp for this element. (**20 points**)

$$a_1 = V_1 + z_0 i_1 \; ; \quad a_2 = V_2 + z_0 i_2 ; \quad b_1 = V_1 - z_0 i_1 \; ; \quad b_2 = V_2 - z_0 i_2$$

$$\therefore \quad V_1 - z_0 i_1 = s_{11}(V_1 + z_0 i_1) + s_{12}(V_2 + z_0 i_2)$$
$$(V_2 - z_0 i_2) = s_{21}(V_1 + z_0 i_1) + s_{22}(V_2 + z_0 i_2)$$

So we have: $V_1(1-s_{11}) - z_0(1+s_{11})i_1 - s_{12}V_2 - s_{12}z_0 i_2 = 0$

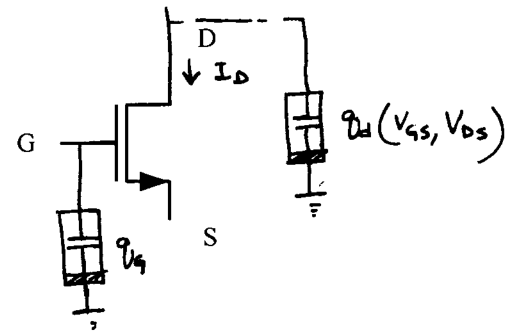
$$-s_{21}V_1 - z_0 s_{21} i_1 + V_2(1-s_{22}) - z_0(1+s_{22})i_2 = 0$$

$$V_1 = V_A - V_B \; ; \qquad V_2 = V_C - V_D$$

The stamp is

$$\begin{array}{c} \\ A \\ B \\ C \\ D \\ BCR1 \\ BCR2 \end{array}
\begin{array}{c} \begin{array}{cccccc} V_A & V_B & V_C & V_D & i_1 & i_2 \end{array} \\
\begin{bmatrix} & & & & 1 & \\ & & & & -1 & \\ & & & & & 1 \\ & & & & & -1 \\ (1-s_{11}) & -(1-s_{11}) & -s_{12} & s_{12} & -z_0(1+s_{11}) & -z_0 s_{12} \\ -s_{21} & s_{21} & (1-s_{22}) & -(1-s_{22}) & -z_0 s_{21} & -z_0(1+s_{22}) \end{bmatrix} \end{array}
\quad
\begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \text{RHS} \end{array}$$

2. Assume that the MOSFET is a 3 terminal device with $I_d = i(V_{gs}, V_{ds})$. Furthermore, assume that the charge in the device can be represented as two nonlinear capacitors from the drain and gate nodes, respectively to ground. The charge for these capacitors is described by (drain) $Q_d = q_d(V_{gs}, V_{ds})$, and (gate) $Q_g = q_g(V_{gs}, V_{ds})$. Use this information to write the Jacobian matrix and RHS stamp for timepoint n at Newton iteration $k+1$. Assume the integration method is described by $\dot{x}_n = \alpha x_n + \beta$. **(35 points)**



$$I_D = i(V_{gs}, V_{ds})$$

$$g_m = \frac{\partial I_D}{\partial V_{GS}} = \frac{\partial i}{\partial V_{GS}} \; ; \; g_{ds} = \frac{\partial I_D}{\partial V_{DS}}$$

For charge apply integration method:

$$i = \frac{dq}{dt} \Rightarrow i_n = \alpha q_n + \beta_q$$

$$i_{n, q_d} = \alpha\, q_d(V_{GS}, V_{DS}) + \beta q_d$$

$$i_{n, q_d}^{k+1} = \alpha q_d^k + \alpha \underbrace{\frac{\partial q_d}{\partial V_{GS}}}_{C_{dg}} (V_{GS}^{k+1} - V_{GS}^k) + \alpha \underbrace{\frac{\partial q_d}{\partial V_{DS}}}_{C_{dd}} (V_{DS}^{k+1} - V_{DS}^k) + \beta q_d$$

$$C_{gd} = \partial q_g / \partial V_{DS} \; ; \; C_{gg} = \partial q_G / \partial V_{GS}$$

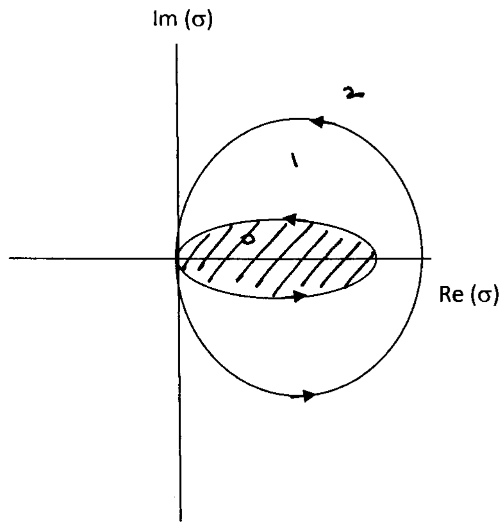
The stamp at iteration $k+1$ is then

$$\begin{array}{c} \\ D \\ G \\ S \end{array} \begin{array}{c} \begin{array}{ccc} \quad D \quad & \quad G \quad & \quad S \quad \end{array} \\ \begin{bmatrix} g_{ds} + \alpha C_{dd} & g_m + \alpha C_{dg} & -g_{ds} - g_m - \alpha C_{dd} - \alpha C_{dg} \\ \alpha C_{gd} & \alpha C_{gg} & -\alpha C_{gd} - \alpha C_{gg} \\ -g_{ds} & -g_m & g_m + g_{ds} \end{bmatrix} \end{array} \quad \text{Jacobian}$$

$$\begin{array}{c} D \\ G \\ S \end{array} \begin{bmatrix} -I_K - \alpha q_d^k + \alpha C_{dd} V_{DS}^k + \alpha C_{dg} V_{GS}^k - \beta q_d \\ -\alpha q_G^k + \alpha C_{gd} V_{DS}^k + \alpha C_{gg} V_{GS}^k - \beta q_G \\ + I_K \end{bmatrix} \quad \text{RHS}$$

where $I_K = I_D^k - g_m V_{GS}^k - g_{DS} V_{DS}^k$

3. The Γ_σ contour for an integration method is shown below. Shade the region of absolute stability. (**10 points**)



4. A linear multistep integration method is given by

$$x_n = x_{n-1} + \frac{h}{2\theta}\dot{x}_n + \frac{h}{2\theta}(2\theta - 1)\dot{x}_{n-1}$$

$\alpha_0 = 1, \ \alpha_1 = -1$

$\beta_0 = -\frac{1}{2\theta}, \ \beta_1 = -\frac{(2\theta-1)}{2\theta}$

a) For what values of θ is this a first-order integration method. (**5 points**) $\sum \beta_i = -1$

$K=0: \quad \sum \alpha_i = 0 \qquad 1 + (-1) = 0$

$K=1: \quad (-1)\alpha_1 + 1(\beta_0 + \beta_1) = (-1)(-1) + 1(-1) = 0$

$K=2: \quad (-1)^2\alpha_1 + 2[\beta_1(-1)] = -1 + 2\left[\frac{2\theta-1}{2\theta}\right] = -1 + 2 - \frac{1}{\theta} = 1 - \frac{1}{\theta}$

For first-order $K=2 \neq 0 \Rightarrow 1 - \frac{1}{\theta} \neq 0$ or $\theta \neq 1$

b) For what values of θ is this a second-order integration method. (**5 points**)

$K=3: \quad (-1)^3\alpha_1 + 3[\beta_1(-1)^2] = (-1)(-1) + 3\left[-\frac{(2\theta-1)}{2\theta}\right]$

$= 1 - 3 + \frac{3}{2\theta} = -2 + \frac{3}{2\theta}$

For second-order $K=2 = 0$ & $K=3 \neq 0$

$K=2 = 0$ for $\theta = 1$ & $K=3 \Rightarrow -2 + \frac{3}{2} \neq 0$

$\therefore \ \theta = 1 \Rightarrow$ 2nd order method (Note This is TR)

c) For the condition of part (a) what is the local error. (**7 points**) $\sum \beta_i = -1$ so already normalized

$\theta \neq 1$ Use $k=2$ term to get c_2

$$c_2 = \left(1 - \frac{1}{\theta}\right)\frac{1}{2!}$$

$$\therefore \quad LE = \frac{1}{2}\left(1 - \frac{1}{\theta}\right) h^2 \ddot{x}(t_n)$$

d) For the condition of part (b) what is the local error. (**8 points**)

$\theta = 1$: Use $k=3$ term to get c_3

$$c_3 = \frac{1}{3!}\left(-2 + \frac{3}{2}\right) = \frac{1}{6}\left(-\frac{1}{2}\right) = -\frac{1}{12}$$

$$\therefore \quad LE = -\frac{1}{12} h^3 \dddot{x}(t_n)$$

e) Write the equation for the region of absolute stability for this integration method. ***You don't need to solve the equation or draw a plot.*** (**10 points**)

For stability use the test problem $\dot{x} = \lambda x$

$$x_n - x_{n-1} - \frac{h}{2\theta}\dot{x}_n - \frac{h}{2\theta}(2\theta - 1)\dot{x}_{n-1} = 0$$

$$z - 1 - \frac{\sigma}{2\theta} z - \frac{\sigma}{2\theta}(2\theta - 1) = 0 \qquad \sigma = \lambda h$$

$$z\left(1 - \frac{\sigma}{2\theta}\right) = 1 + \frac{\sigma}{2\theta}(2\theta - 1)$$

$$\text{or} \quad z = \frac{1 + \frac{\sigma}{2\theta}(2\theta - 1)}{1 - \frac{\sigma}{2\theta}} = \frac{2\theta + \sigma(2\theta - 1)}{2\theta - \sigma}$$

For stability require $|z| \leq 1 \Rightarrow \left|\frac{2\theta + \sigma(2\theta - 1)}{2\theta - \sigma}\right| \leq 1$