ECE 521 Fall 2016

Test (11/21/16)

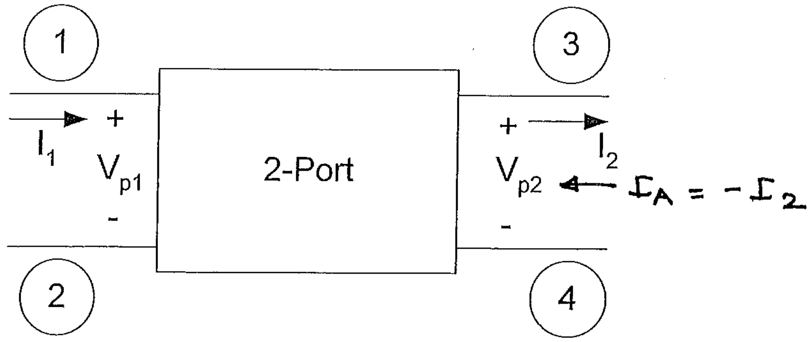
Total # Pages 7
Total # Problems 4

Name SOLUTION ____________________

1.	(20 points)	________
2.	(30 points)	________
3.	(20 points)	________
4.	(30 points)	________
Total	(100 points)	________

GOOD LUCK

1. The two-port shown below is defined in terms of *ABCD* or *transmission* parameters.



The *ABCD*-parameters are defined as (note the direction of I_2)

$$V_{p1} = AV_{p2} + BI_2$$

$$I_1 = CV_{p2} + DI_2$$

Write the smallest MNA stamp (matrix and RHS) for this element. **(20 points)**

For nodal eqns current leaving node is positive

$$\therefore \; I_A = -I_2$$

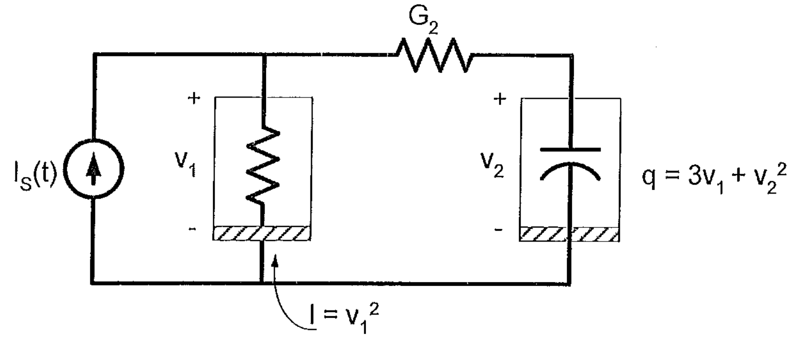
$$\Rightarrow \; V_{P1} = V_1 - V_2 = A(V_3 - V_4) - B I_A$$

$$I_1 = C(V_3 - V_4) - D I_A$$

For the smallest MNA stamp use I_A as the unknown current

$$\begin{array}{l c c}
 & \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & I_A \end{array} & \text{RHS} \\
\begin{array}{l} \text{KCL@1} \\ \text{KCL@2} \\ \text{KCL@3} \\ \text{KCL@4} \\ \text{BCR} \end{array} & \begin{bmatrix} & & +C & -C & -D \\ & & -C & +C & +D \\ & & & & +1 \\ & & & & -1 \\ 1 & -1 & -A & +A & +B \end{bmatrix} & \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
\end{array}$$

2. For the circuit shown below answer the following questions:



a) Write the equations that describe the dynamic behavior of the circuit. Take q, the capacitor branch charge, as an unknown. **(5 points)**

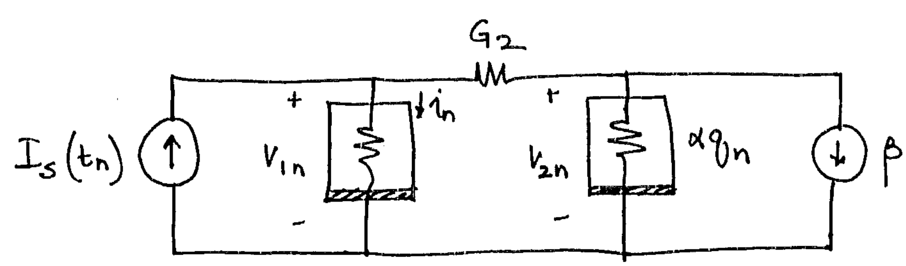
$$V_1^2 + G_2(V_1 - V_2) = I_s(t)$$

$$G_2(V_2 - V_1) + \frac{dq}{dt} = 0$$

$$-q + 3V_1 + V_2^2 = 0$$

b) Draw the companion model for the complete circuit at timepoint t_n. Assume an integration method described by $\dot{x}_n = \alpha x_n + \beta$ is used. **(5 points)**

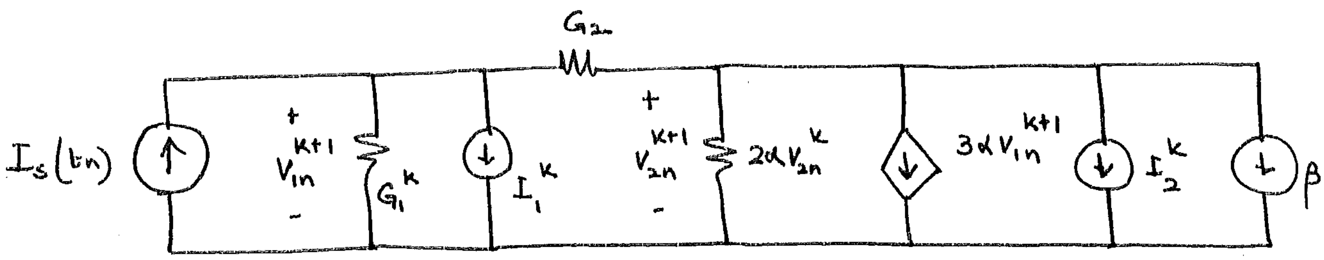
$$\dot{x}_n = \alpha x_n + \beta \Rightarrow \dot{q}_n = \alpha q_n + \beta$$



$$i_n = V_{1n}^2$$

$$q_n = 3V_{1n} + V_{2n}^2$$

c) Draw the companion model for the complete circuit at timepoint t_n and iteration $k+1$ when Newton's method is used to solve the nonlinear equations. **(10 points)**



$$G_1^k = 2V_{1n}^k$$

$$I_1^k = i_n(V_{1n}^k) - (2V_{1n}^k)V_{1n}^k$$

$$I_2^k = \alpha q(V_n^k) - 3\alpha V_{1n}^k - (2\alpha V_{2n}^k)V_{2n}^k$$

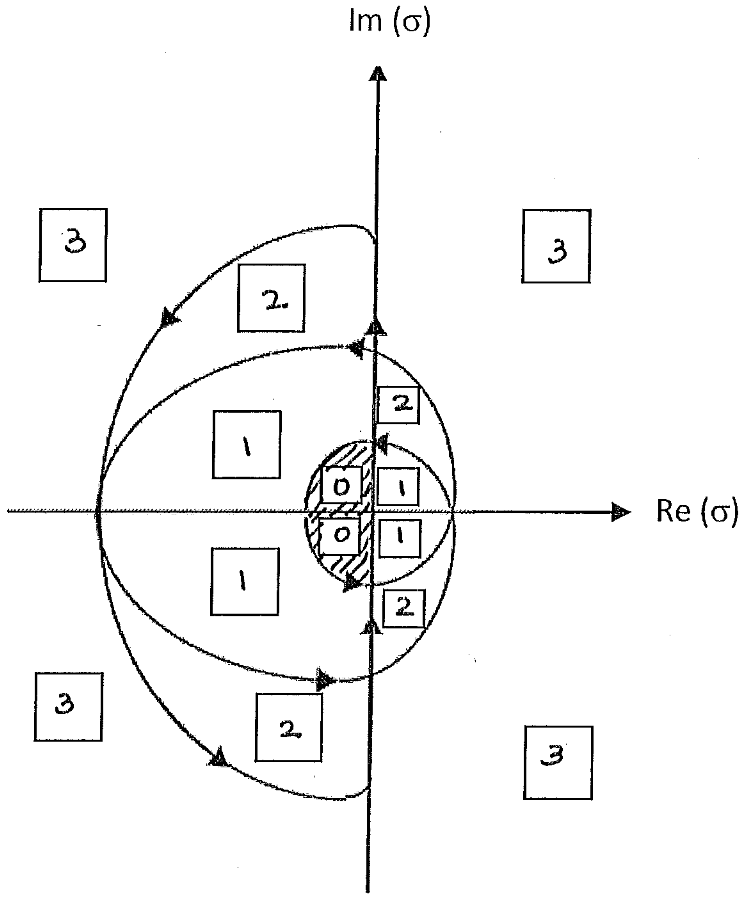
d) Write the Jacobian matrix for this circuit at Newton iteration $k+1$. Take q to be an unknown. **(10 points)**

$$\begin{array}{cc} & \begin{array}{ccc} V_1 & V_2 & q \end{array} \\ \begin{array}{c} KCL\,@\,1 \\ KCL\,@\,2 \\ BCR\text{-}q \end{array} & \begin{bmatrix} 2V_{1n}^k + G_2 & -G_2 & 0 \\ -G_2 & +G_2 & \alpha \\ 3 & 2V_{2n}^k & -1 \end{bmatrix} \end{array}$$

OR

$$\begin{array}{cc} & \begin{array}{cc} V_1 & V_2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 2V_{1n}^k + G_2 & -G_2 \\ -G_2 + 3\alpha & +G_2 + 2\alpha V_{2n}^k \end{bmatrix} \end{array}$$

3. The Γ_σ contour for a 7^h order integration method is shown below. In each box fill in the number of roots outside the unit circle ($|z| = 1$) and shade the region of absolute stability. **(20 points)**



4. A linear multistep integration method is given by

$$x_n = x_{n-1} + \alpha h\dot{x}_n + \beta h\dot{x}_{n-1} \qquad 0 \le \beta \le \frac{1}{2}$$

a) What is the relation between β and α for the given method to be a first-order integration method? **(5 points)**

$$x_n - x_{n-1} - \alpha h\dot{x}_n - \beta h\dot{x}_{n-1} = 0$$

$$\Rightarrow \alpha_0 = 1, \quad \alpha_1 = -1, \quad \beta_0 = -\alpha, \quad \beta_1 = -\beta$$

Exactness constraints:

$$\alpha_0 + \alpha_1 = 1 - 1 = 0 \quad \text{satisfied}$$

$$-\alpha_1 + \beta_0 + \beta_1 = 0 \quad = 1 - \alpha - \beta = 0 \quad \text{or } \alpha + \beta = 1$$

$$\alpha_1 + 2\beta_1(-1) \neq 0 \Rightarrow -1 + 2\beta \neq 0 \Rightarrow \beta \neq \frac{1}{2}$$

$\therefore$ condition is $\alpha = 1 - \beta$, $\beta \neq \frac{1}{2}$

or $0 \le \beta < \frac{1}{2}$

b) What is the relation between β and α for the given method to be a second-order integration method? **(5 points)**

For 2nd order require

$$\alpha_1 + 2\beta_1(-1) = 0 \Rightarrow -1 + 2\beta = 0 \quad \text{or } \beta = \frac{1}{2}$$

Check: $-\alpha_1 + 3\beta_1 = 1 - 3\beta \neq 0$

$\therefore$ 2nd order for $\beta = \frac{1}{2} \Rightarrow \alpha = \frac{1}{2}$

i.e. TR method

c) For the condition of part (a) what is the local error. **(10 points)**

First-order method $\alpha = 1-\beta$, $\beta \neq \frac{1}{2}$

$$LE = \frac{c_2}{2} h^2 \ddot{x}(t_n)$$

where $c_2 = \alpha_1 - 2\beta_1$ Since $\sum_{i=0}^{1} \beta_i = \beta_0 + \beta_1$

$= -1 + 2\beta$ $= -\alpha - \beta = -1$

i.e. already normalized

$$\therefore LE = -\frac{1+2\beta}{2} h^2 \ddot{x}(t_n)$$

or $-\frac{(1-2\beta)}{2} h^2 \ddot{x}(t_n$ Since $\beta < \frac{1}{2}$

d) For the condition of part (a) and $\beta = 1/4$, write the equation for the region of absolute stability for the given method. ***You don't need to solve the equation or draw a plot.*** **(10 points)**

$$\beta = \frac{1}{4} \Rightarrow \alpha = 1 - \beta = \frac{3}{4}$$

$$\Rightarrow x_n - x_{n-1} - \frac{3}{4} h \dot{x}_n - \frac{1}{4} h \dot{x}_{n-1} = 0$$

$$\dot{x} = \lambda x$$

$$\Rightarrow x_n - x_{n-1} - \frac{3}{4} h\lambda x_n - \frac{1}{4} h \lambda x_{n-1} = 0$$

$$\left(1 - \frac{3}{4}\sigma\right) x_n - \left(1 + \frac{1}{4}\sigma\right) x_{n-1} = 0 \qquad \sigma = \lambda h$$

$$\text{or } z = \frac{1 + \frac{\sigma}{4}}{1 - \frac{3}{4}\sigma} = \frac{4+\sigma}{4-3\sigma}$$

require $|z| \leq 1$ $\Rightarrow \left|\frac{4+\sigma}{4-3\sigma}\right| \leq 1$

(single root)