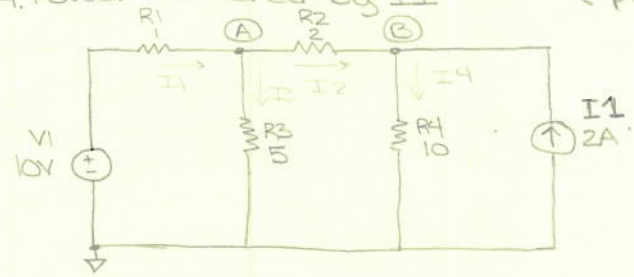
KCL Problems

7. For the following circuit, use KCL to find:

1. V_A (3pts)
2. V_B (3pts)
3. Power delivered by V1 (2pts)
4. Power delivered by I1 (2pts)



KCL ⇒ Σ current in = Σ current out

Write currents in terms of voltages

$I1 = \frac{10 - V_A}{1}$ (R1) $\quad I2 = \frac{V_A - V_B}{2}$ (R2) $\quad I3 = \frac{V_A}{5}$ (R3) $\quad I4 = \frac{V_B}{10}$

Node A

$I1 = I3 + I2$

$\left(\frac{10 - V_A}{1} = \frac{V_A}{5} + \frac{V_A - V_B}{2}\right)10$

$10(10 - V_A) = 2V_A + (V_A - V_B)5$

$100 - 10V_A = 2V_A + 5V_A - 5V_B$

$100 - 17V_A = -5V_B$

$V_B = \frac{17V_A - 100}{5}$

Node B

$I2 + 2 = I4$

$\left(\frac{V_A - V_B}{2} + 2 = \frac{V_B}{10}\right)10$

$5(V_A - V_B) + 20 = V_B$

$5V_A - 5V_B + 20 = V_B$

$5V_A + 20 = 6V_B$

$V_B = \frac{5V_A + 20}{6}$

Set two equations for V_B equal

$\frac{17V_A - 100}{5} = \frac{5V_A + 20}{6}$ clear all fractions

$5(5V_A + 20) = 6(17V_A - 100)$

$25V_A + 100 = 102V_A - 600$

$700 = 77V_A$

$V_A = \frac{700}{77} = \boxed{9.09V}$ ✓

Solve for V_B

$V_B = \frac{5V_A + 20}{6}$

$V_B = \frac{5(9.09) + 20}{6}$

$\boxed{V_B = 10.909V}$ ✓

Power delivered by V1 (positive number)

$P = IV$ where $I = -I1$

$P = (0.91)(10)$

$\boxed{P = 9.1\ W \text{ delivered}}$ ✓

Power delivered by I1 (positive also)

$P = IV$ where $V = V_B - 0$

$P = (2)(10.909)$

$\boxed{P = 21.818\ W \text{ delivered}}$ ✓