

## The dB scale And Neper

Long Ago, telephone companies measured power loss on telephone cables with a unit of measure called "mile of standard cable". This measure was used until 1923. It was a simple way to compare efficiency or loss on a telephone cable. The "MSG" is a ratio between the powers of an 800hz signal at the ends of a one mile length of standard cable.

In 1929, a logarithmic unit called the "bel" was adopted as the international unit of power ratio. It was named after Alexander Graham Bell. The bel and more importantly the decibel are now used to express the ratio between 2 values with identical dimensions. We can express ratios in dB for power, voltage, sound pressure, etc. Since the dimensions cancel, the decibel is dimensionless.

Bels differ by  $10^{N(1)}$ , decibels by  $10^{N(0.1)}$

$$\text{difference in decibels} = 10 \log_{10} \left( \frac{\text{Value 1}}{\text{Value 2}} \right) (\text{dB})$$

When expressing power differences, the human ear is just able to distinguish a 1 dB difference in sound pressure level.

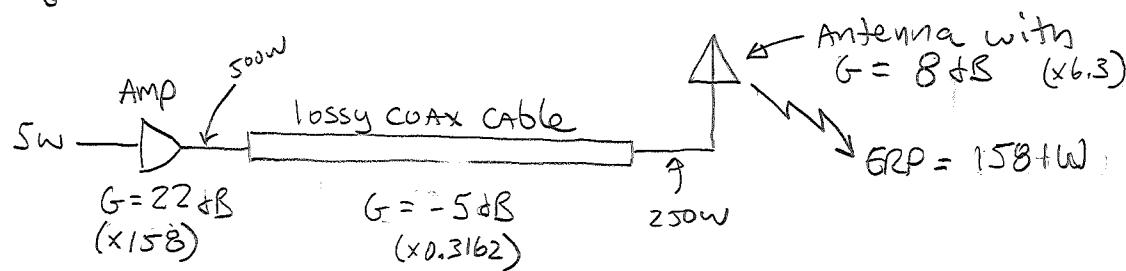
## The dB scale and Neper

The dB operator acts as a scale converter of like relative quantities, such as  $\frac{\text{Power Out}}{\text{Power In}}$ .

If power gain is  $G$ , then  $G = \frac{P_{\text{out}}}{P_{\text{in}}}$  And power gain in dB is  $G_{\text{dB}} = 10 \log \left( \frac{P_{\text{out}}}{P_{\text{in}}} \right)$

Gain or loss of electronic devices can vary over many orders of magnitude. The utility of the decibel scale comes in its logarithmic nature. For example, if gain as shown above varies by six orders of magnitude from  $10^{-3}$  to  $10^{+3}$ , the gain in dB varies from  $-30$  to  $+30$  dB.

Also, if we express the gain of amplifiers in dB instead of multiplicative factors, determining the gain OR loss in a cascade of them is reduced to simply adding their gain in dB.



The power gain through the Amp, COAX and Antenna is  $(22 - 5 + 8) = 25 \text{ dB}$

This is easier to compute than  $(158 \times 0.3162 \times 6.3)$ .

The output power is:  $5 \times 158 \times 0.3162 \times 6.3 = 1581 \text{ W}$

## The dB scale And Neper

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We can reference absolute values (not ratios) if we define a reference value. For example,

dBm : ratio in dB referenced to 1mW (very popular unit of measure in RF ckt's)

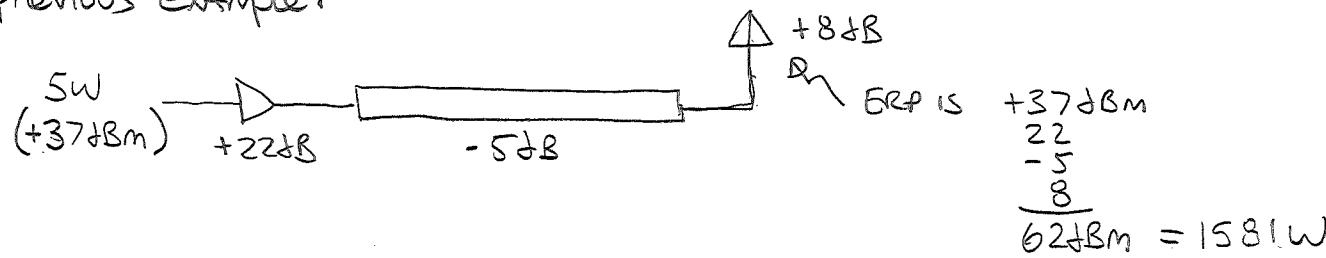
$$\text{dBm} = 10 \log \left( \frac{\text{Power mw}}{1\text{mw}} \right) ; \text{ ex. } 0\text{dBm} = 1\text{mW}, 10\text{dBm} = 10\text{mW}, 20\text{dBm} = 100\text{mW}$$

every 3dB increase doubles power,

6dB quadruples power,

10dB is 10x the original power

Our previous example:



Computing power, or gain or loss can be greatly simplified using the dB scale.

## The dB Scale And Neper

When comparing current or voltage the scaling factor is 20 instead of 10,

$$G_{dB} = 20 \log\left(\frac{V_o}{V_i}\right)$$

$$\text{Since } P = \frac{V^2}{R};$$

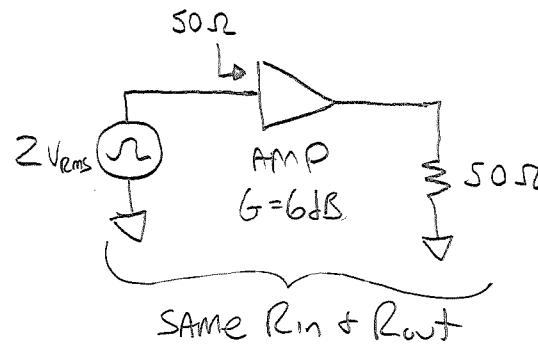
$$\begin{aligned} G_{dB} &= 10 \log_{10} \left( \frac{P_o}{P_i} \right) \\ &= 10 \log_{10} \left( \frac{\frac{V_o^2}{R}}{\frac{V_i^2}{R}} \right) \\ &= 10 \left[ \log_{10}(V_i^2) - \log_{10}(V_o^2) \right] \quad \left. \right\} \text{ note that this step requires that both source + load resistances are the same.} \\ &= 10 \left[ 2 \log_{10}(V_i) - 2 \log_{10}(V_o) \right] \\ &= 20 \log_{10} \left( \frac{V_i}{V_o} \right) \end{aligned}$$

# The dB Scale and Neper

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Gain in dB using voltage OR power are equivalent only if the resistance at both input and output are the same.

For example,

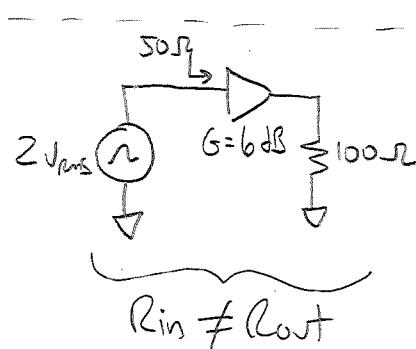


$$\text{Gain in dB using Volts: } \text{dB} = 20 \log_{10} \left( \frac{V_o}{V_i} \right)$$

$$6 = 20 \log_{10} \left( \frac{V_o}{2} \right)$$

$$2 = \frac{V_o}{2} \Rightarrow V_o = 4V_{pp}$$

every 6dB  
Voltage gain  
doubles voltage



$$\text{Gain in dB using power: } \text{dB} = 10 \log_{10} \left( \frac{P_o}{P_i} \right)$$

$$\text{Volts: } \text{dB} = 20 \log_{10} \left( \frac{V_o}{2} \right)$$

$$V_o = 4V_{pp} \quad \text{same as before}$$

$$\text{power: } \text{dB} = 10 \log_{10} \left( \frac{P_o}{P_i} \right)$$

$$P_i = 80 \text{ mW} \quad P_o = \frac{16}{100} = 160 \text{ mW}$$

$$\text{dB} = 10 \log \left( \frac{160}{80} \right) = \underline{\underline{3 \text{dB}}} \\ \underline{\underline{\neq 6 \text{dB}}}$$

$$P_i = \frac{z^2}{50} = 80 \text{ mW}, \quad P_o = \frac{4^2}{50} = 320 \text{ mW}$$

$$\text{dB} = 10 \log \left( \frac{0.32}{0.08} \right) = 6.02 \text{ dB}$$

every 3dB doubles power, 6dB is 4x

## The dB Scale And Neper

The Neper is another dimensionless unit used to express the ratio between two quantities. It is very similar to the dB except that it uses the natural logarithm.

For voltages:

$$dB = 20 \log_{10} \left( \frac{V_o}{V_i} \right)$$

$$Np = \ln \left( \frac{V_o}{V_i} \right)$$

The difference between Neper and dB is a fixed ratio.

$$\frac{8.686dB}{Np} \quad \text{And} \quad \frac{0.115Np}{dB}$$

The Neper was named after John Neper, the inventor of logarithms