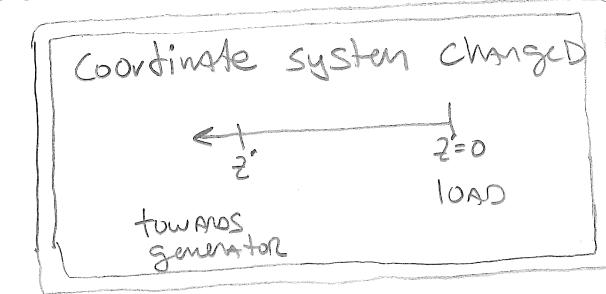


IMPEDANCE TRANSFORMATION

The general solution to the complex wave equation: $\frac{d^2V(z)}{dz^2} = -(\omega^2 LC) V(z)$

is of the form: $V(z) = V_+ e^{+j\beta z'} + V_- e^{-j\beta z'}$

total voltage }
 phasor at any }
 point z'
 FORWARD }
 traveling }
 wave
 (towards load) + reflected
 wave
 (towards generator)



Then the current can be expressed as:

$$I(z) = \frac{1}{Z_0} [V_+ e^{+j\beta z'} - V_- e^{-j\beta z'}] \quad \text{where } Z_0 = \sqrt{\frac{L}{C}}$$

Thus, the impedance seen looking into the T-Line towards the load Z_L
 At Any position along the line is:

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \left[\frac{V_+ e^{+j\beta z'} + V_- e^{-j\beta z'}}{V_+ e^{+j\beta z'} - V_- e^{-j\beta z'}} \right]$$

Clearly, $Z(z)$ consists of both real + imaginary parts. $Z(z) = R(z) \pm jX(z)$

This formula says nothing about Z_L ! $Z(z)$ is expressed in terms of forward + reflected voltages.

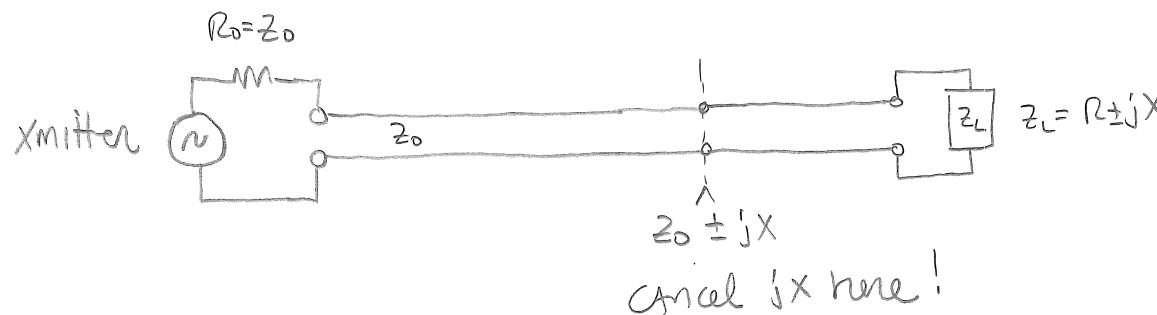
Impedance Transformation

We are beginning to search for ways to find $z_{in}(z)$ first at the input to the line and then later at arbitrary points of z . Why?

In the Analog domain (RF) we want to deliver maximum power to some load. But the load is often reactive.

We want the transmitter to look into a SOSR resistive load.

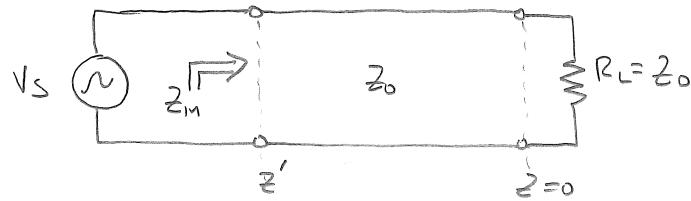
- Can we place our transmitter at the right place on the line? Magic spot?
- even better, can we find a place on the line to connect reflections at the transmitter?
- can we find a place on the line where the rest portion of the impedance is equal to z_0 ?



Impedance Transformation

Look at some special cases for Z_{in} . $Z_L = Z_0$, $Z_L = \emptyset$, $Z_L = \infty$

Line terminates in Z_0 ($Z_L = Z_0$) (A "flat" line)



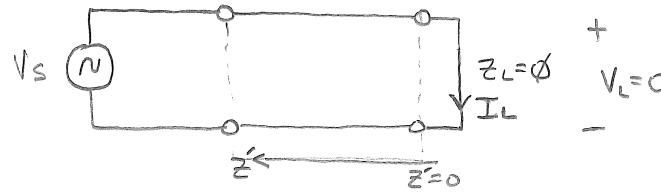
Since $R_L = Z_0$, $\sqrt{-} = 0$; so

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \left[\frac{V^+ e^{j\beta z'} + V^- e^{-j\beta z'}}{V^+ e^{j\beta z'} - V^- e^{-j\beta z'}} \right] \xrightarrow{\text{(no reflection)}} Z_0$$

this agrees with our prior understanding of matched losses

Impedance Transformation

Line Terminated in Short Circuit $z_L = 0$



$$\text{From before: } V(z') = V^+ e^{+j\beta z'} + V^- e^{-j\beta z'}$$

At the load: ($z' = 0$) $V_L = [V(z')]_{z'=0} = [V^+ e^{+j\beta z'} + V^- e^{-j\beta z'}]_{z'=0} = V^+ + V^- = 0$ Agrees w/circuit
(sanity check)

$$I_L [I(z')]_{z'=0} = \frac{1}{Z_0} [V^+ - V^-] = \frac{1}{Z_0} [V^+ - (-V^+)] = \frac{2V^+}{Z_0}$$

↗ reflection is incident $\neq -1$

familiar?

Anywhere else on the line:

$$\begin{aligned} V(z') &= V^+ e^{+j\beta z'} - V^+ e^{-j\beta z'} \quad (\text{reflection is } -1 \text{ times incident}) \\ &= V^+ (e^{j\beta z'} - e^{-j\beta z'}) \quad \left\{ e^{-ix} - e^{ix} = -2i \sin(x) \right\} \\ &= -2V^+ j \sin(\beta z') \end{aligned}$$

$$I(z') = \frac{V^+}{Z_0} [e^{+j\beta z'} + e^{-j\beta z'}] = \frac{2V^+}{Z_0} \cos(\beta z') \quad \left\{ e^{-ix} + e^{ix} = 2 \cos(x) \right\}$$

Solving for $z(z')$

$$z(z') = \frac{V(z')}{I(z')} = Z_0 \left(\frac{-2jV^+ \sin(\beta z')}{2V^+ \cos(\beta z')} \right) = \underline{jZ_0 \tan(\beta z')}$$

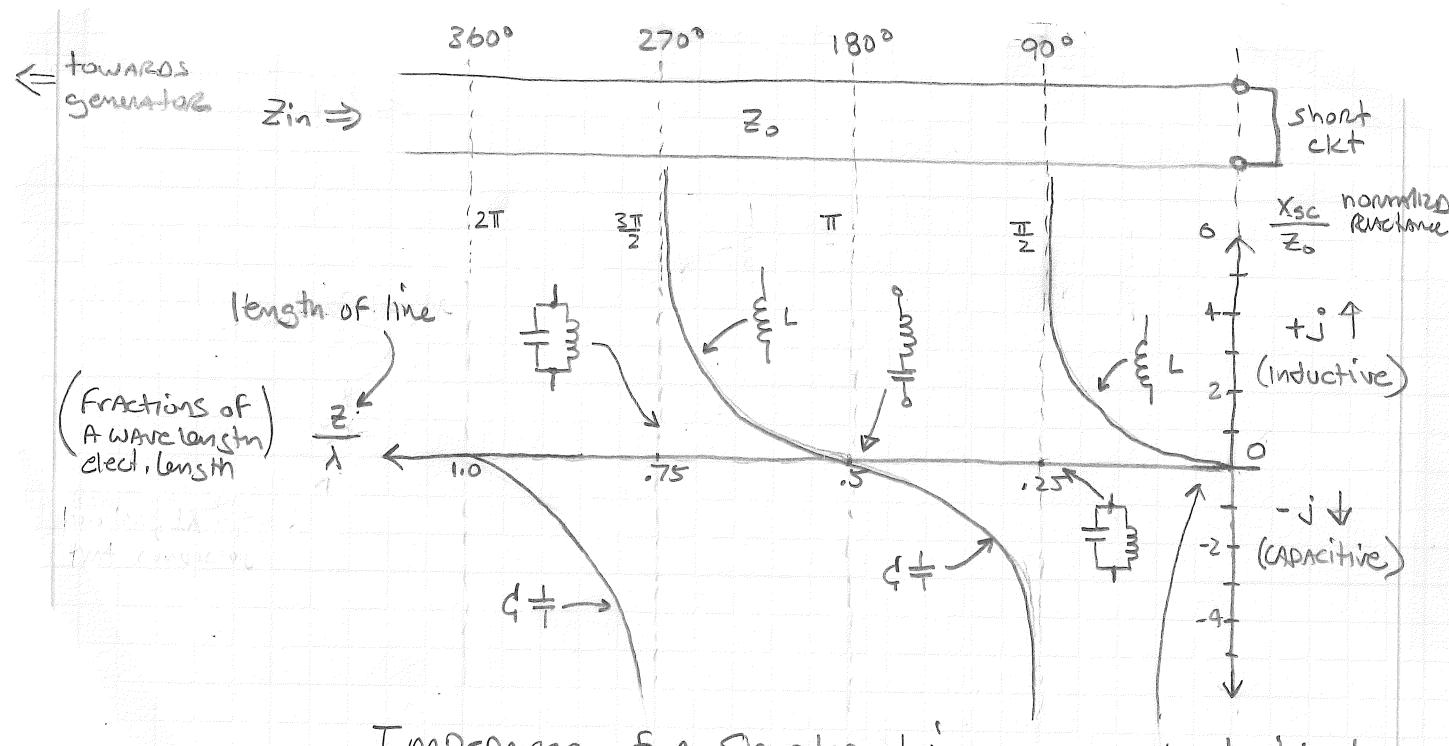
impendence looking into a
shorted T-line at point z'
where $\beta = \frac{2\pi}{\lambda}$

Impedance Transformation

(line terminates in $z_L = 0$)

For the shortest line we have: $z(z') = j z_0 \tan(\beta z')$

We can change z_{in} by changing wavelength or frequency. z can also be either inductive or capacitive... but never resistive. z_{in} will always be purely reactive with a line shorted at the load.



IMPEDANCE of A Shorted Line

$$z_{in}(z') = j Z_0 \tan(2\pi \frac{z'}{\lambda})$$

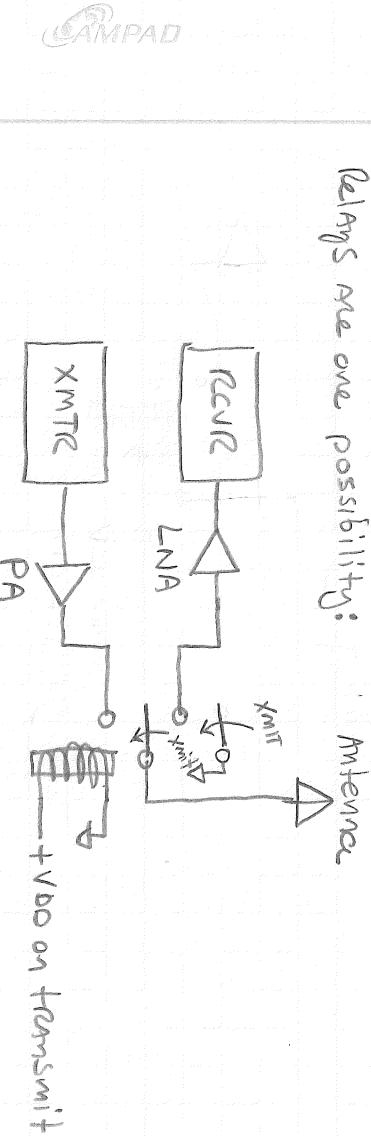
$$\left(\text{number}, \beta = \frac{2\pi}{\lambda} \right)$$

so short, it just looks like a piece of wire

Applications of shorted/open $\lambda/4$ lines.

An RF transceiver usually shares one antenna. Thus some way of switching the transmitter & receiver between the antenna must be devised.

Relays are one possibility:

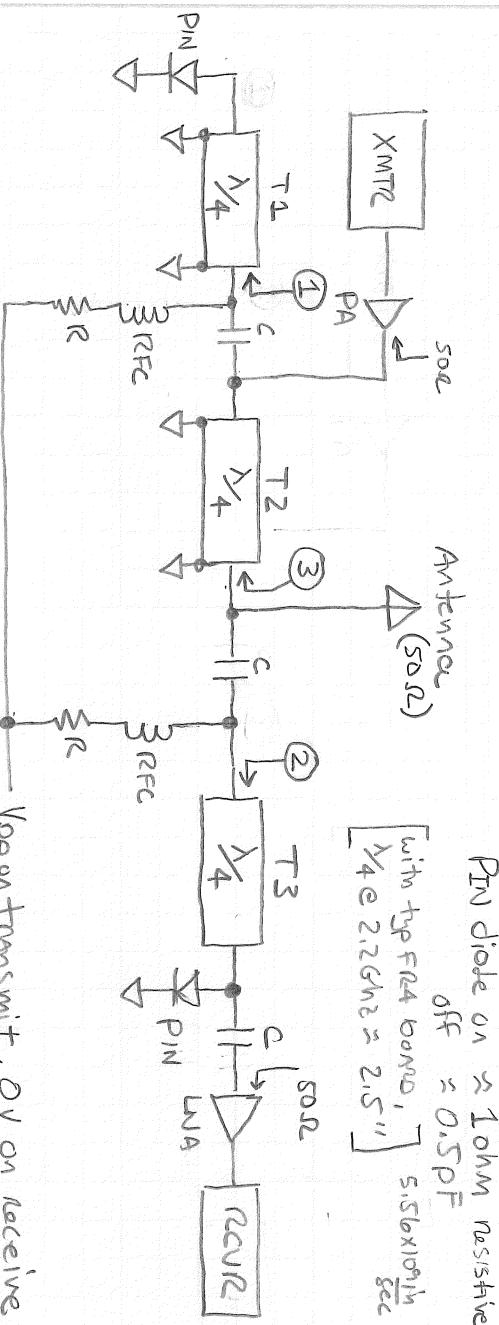


Drawbacks include: Reliability

Speed of changeover

Power required for energizing coil

$\lambda/4$ T-lines are another solution:



CAPACITORS ARE FOR DC BLOCKING + $\approx 0 \times c$ @ FREQUENCY OF INTEREST

RFC = RADIO FREQUENCY CHOICE

PIN = P, INTRINSIC, N

When transmitting, both diodes provide a short to ground. Thus, ① and ② are high-Z. XMT has path to antenna through T2.

When receiving, both diodes are off. Thus ① is shorted to ground, and ③ is high-Z. Rcvr has path to antenna through T3.

PIN diodes act like reverse diodes at lower frequencies but at higher frequencies act as very linear resistors. The large amount of stored charge in the intrinsic region makes the diode unable to rectify at higher frequencies.

Impedance Transformation (line terminatio in $Z_L=0$)

Given 10 cm length of 300Ω twin lead T-line that is shorted at one end,
Find its input impedance @ 300 MHz + X_L . Assume $V_p = c$.

Find electrical length: dielectric is Air; $V_p = 300 \times 10^6$ m/s so $\lambda = \frac{c}{f} = 1$ meter

10 cm is 0.1 meter, so electrical wavelength is 0.1λ

$$\text{so } Z = jZ_0 \tan(\beta z) = j300 \tan\left(2\pi \frac{z}{\lambda}\right)$$

$$= j300 \tan(1628) = \underline{\underline{j218}} \text{ (An inductor)}$$

Since $X_L = 2\pi f L$

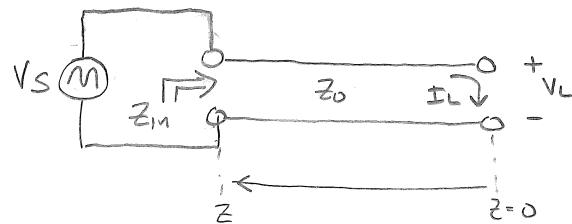
$$L = \frac{218}{(300 \times 10^6) 2\pi} = \underline{\underline{116 \text{nH}}}$$

However, remember that this is not like a 116 nH lumped element.
This T-Line is 116 nH @ 300 MHz + at multiples of that.

How could this be used?

Impedance Transformation

Special case #3: $Z_L = \infty$



From last time $I(z') = \frac{1}{Z_0} [V^+ e^{+j\beta z'} - V^- e^{-j\beta z'}]$; but at the open-circuit end...

$$I_L = [I(z')]_{z'=0} = \frac{1}{Z_0} [V^+ e^{+j\beta z'} - V^- e^{-j\beta z'}]_{z=0} = \frac{1}{Z_0} [V^+ - V^-] = 0 \quad ; \text{ so } V^- = V^+$$

Familiar?

Then, $V(z') = V^+ e^{+j\beta z'} + V^- e^{-j\beta z'}$; And since

$$\begin{aligned} V(z') &= V^+ e^{+j\beta z'} + V^+ e^{-j\beta z'} \\ &= V^+ (e^{+j\beta z'} + e^{-j\beta z'}) = 2V^+ \cos(\beta z) \end{aligned}$$

$$\text{Also, } I(z') = \frac{V^+}{Z_0} (e^{+j\beta z} - e^{-j\beta z}) = -2 \frac{V^+}{Z_0} j \sin(\beta z)$$

$$\text{so } Z(z') = \frac{V(z')}{I(z')} = -j Z_0 \cot(\beta z) \quad \left\{ \begin{array}{l} \text{Impedance looking into an open-circuited} \\ \text{T-line at a point } z' \text{ from the load} \\ \text{where } \beta = \frac{2\pi}{\lambda} \end{array} \right.$$

Once again, we have a purely reactive input impedance.

Impedance Transformation

8

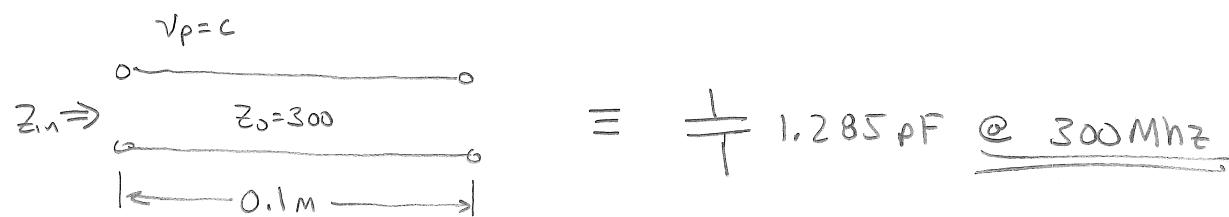
Consider Again the 10cm length of 300Ω transmission line. Now let the end be open circuited. Find $Z_{in}(z)$ And the reactance At 300 Mhz.

$$1\lambda @ 300\text{Mhz} = \frac{300 \times 10^6 \frac{\text{m}}{\text{s}}}{300 \times 10^6 \frac{1}{\text{s}}} = 1 \text{ meter} ; 10\text{cm} = 0.1 \text{ m}$$

$$\begin{aligned} Z_{in}(10\text{cm}) &= -j Z_0 \cot(\beta z) \\ &= -j 300 \cot(2\pi \left(\frac{0.1}{\lambda}\right)) \\ &= -j 300 (1.376) \\ &= -j 413 \Omega \quad (\text{A capacitor}) \end{aligned}$$

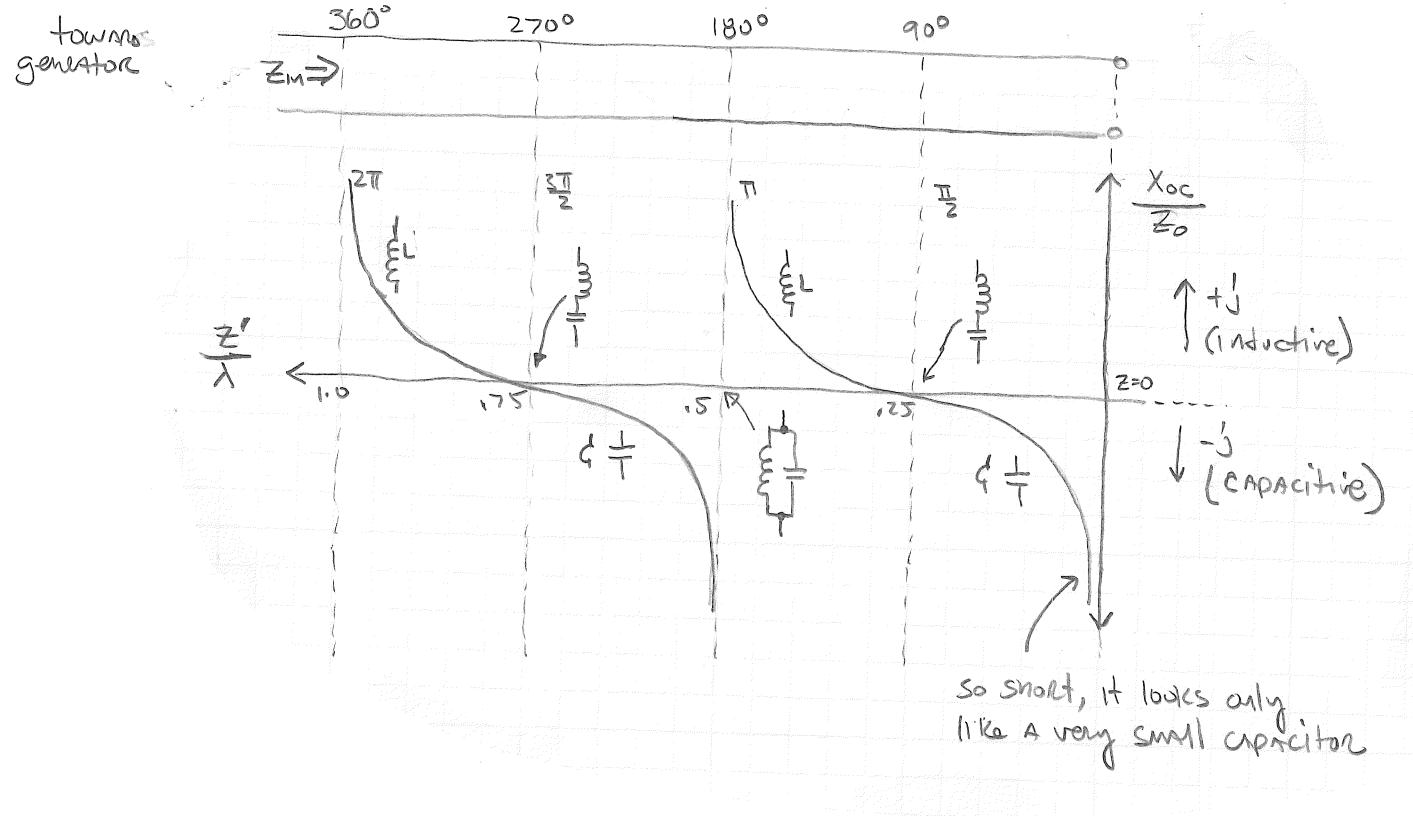
$$X_L = \frac{1}{2\pi f C} ; 413 = \frac{1}{2\pi (300 \times 10^6) C}$$

$$\underline{C = 1.285 \mu\text{F}}$$



Impedance Transformation ($Z_L = \infty$)

So for open-circuited lines:



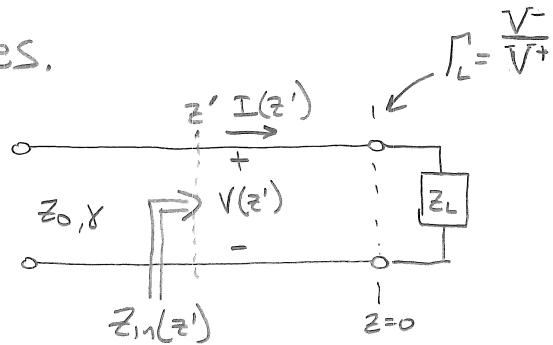
Impedance of A Open Circuited Line

$$Z_{in}(z') = -j Z_0 \cot(2\pi \frac{z'}{\lambda})$$

Impedance Transformation

We can obtain another relationship for $Z_{in}(z')$ that is in terms of $Z_0 + Z_L$ instead of forward + reflected waves.

$$\left. \begin{aligned} V(z') &= V^+ (e^{+\gamma z'} + \Gamma_L e^{-\gamma z'}) \\ I(z') &= \frac{V^+}{Z_0} (e^{+\gamma z'} - \Gamma_L e^{-\gamma z'}) \end{aligned} \right\}$$



$$\begin{aligned} Z_{in}(z') &= Z_0 \left(\frac{e^{+\gamma z'} + \Gamma_L e^{-\gamma z'}}{e^{+\gamma z'} - \Gamma_L e^{-\gamma z'}} \right) \\ &= Z_0 \left(\frac{Z_L + Z_0 \tanh(\gamma z')}{Z_0 + Z_L \tanh(\gamma z')} \right) \quad \text{if lossy} \\ &= Z_0 \left(\frac{Z_L + j Z_0 \tan(\beta z')}{Z_0 + j Z_L \tan(\beta z')} \right) \quad \text{if lossless} \end{aligned}$$

Let's look again at some special cases...

Impedance Transformation

$$Z_{in}(z') = Z_0 \left(\frac{Z_L + jZ_0 \tan(\beta z')}{Z_0 + jZ_L \tan(\beta z')} \right) \quad (\text{lossless line})$$

If $Z_L = Z_0$; $Z_{in}(z') = Z_0$ (At Any z' !)

$$\text{If } Z_L = 0; Z_{in}(z') = Z_0 \left(\frac{\cancel{Z_L}^0 + jZ_0 \tan(\beta z')}{\cancel{Z_0}^0 + jZ_L \tan(\beta z')} \right) \rightarrow 0 = jZ_0 \tan(\beta z'); \text{ just like before}$$

$$\text{If } Z_L = \infty; Z_{in}(z') = Z_0 \left(\frac{\cancel{Z_L}^\infty + jZ_0 \tan(\beta z')}{\cancel{Z_0} + jZ_L \tan(\beta z')} \right) \rightarrow \infty = -jZ_0 \cot(\beta z'); \text{ As before}$$

But what happens if you have a line $\frac{\lambda}{4}$ long? $z' = \frac{\lambda}{4}$

$$\tan(\beta z') = \tan\left(\frac{2\pi}{\lambda} \cdot \frac{\lambda}{4}\right) = \tan\frac{\pi}{2} = \infty; \text{ so}$$

$$\begin{aligned} \text{If } z' = \frac{\lambda}{4}; Z_{in}(z') &= Z_0 \left(\frac{Z_L + jZ_0 + \cancel{\tan(\beta z')}^\infty}{Z_0 + jZ_L + \cancel{\tan(\beta z')}^\infty} \right) && (\text{Any number added to } \infty \text{ is still } \infty) \\ &= Z_0 \left(\frac{jZ_0}{jZ_L} \right) \end{aligned}$$

$$\boxed{Z_{in}(z')} = \frac{Z_0^2}{Z_L}; \text{ hmmm, interesting result, what can you do with it?}$$

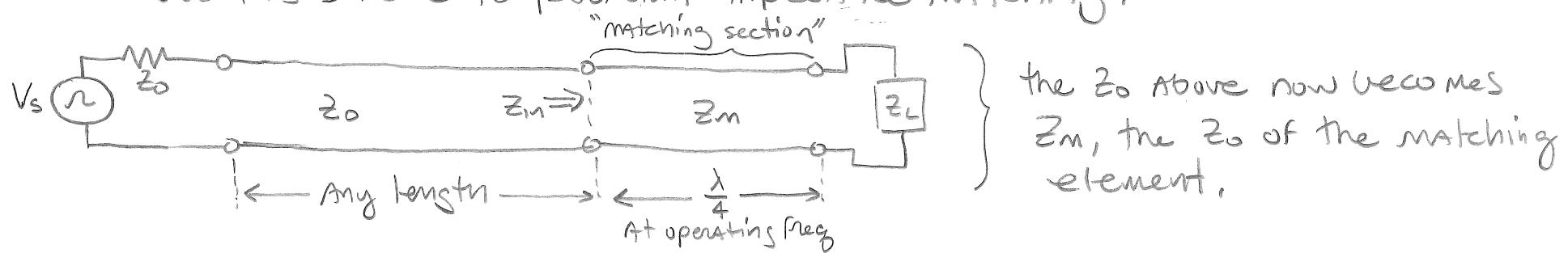
($\frac{\lambda}{4}$ transformation)

From before we have:

$$\left. Z_{in}(z') \right|_{z'=\frac{\lambda}{4}} = \frac{Z_0^2}{Z_L} \quad \left\{ \begin{array}{c} \text{IP} \\ \xrightarrow{Z_{in}} \\ z=\frac{\lambda}{4} \end{array} \right. \quad \left. \begin{array}{c} \xrightarrow{Z_0} \\ \text{or} \\ z=0 \end{array} \right. \quad \left. \begin{array}{c} \boxed{Z_L} \\ \text{or} \\ z=0 \end{array} \right.$$

This scheme transforms the impedance Z_L depending on Z_0 .

Let's use this scheme to perform impedance matching.



We want $Z_{in} = Z_0$. For this network, $Z_{in} = \frac{Z_m^2}{Z_L}$. So then

$$Z_0 = \frac{Z_m^2}{Z_L}$$

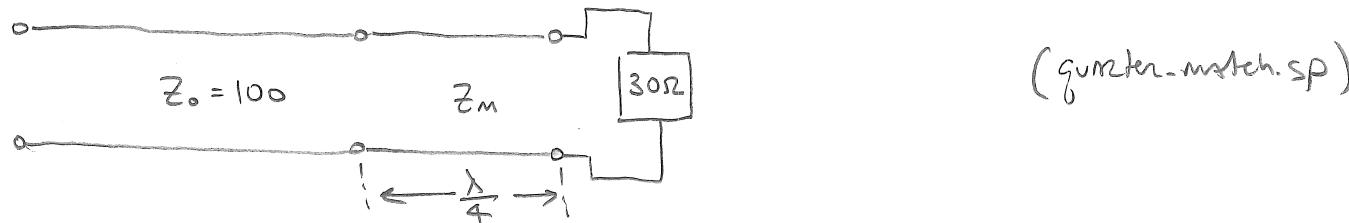
$$Z_m^2 = Z_0 Z_L$$

$$\underline{Z_m = \sqrt{Z_0 Z_L}}$$

$\left. \begin{array}{l} \text{So, if the matching section has a } Z_0 \text{ of } \sqrt{Z_0 Z_L} \text{ and is} \\ \text{ } \lambda/4 \text{ long at operating frequency, then } Z_L \text{ is transformed to } Z_0. \\ \text{Restriction: coax cables present real impedances. So if } Z_m \\ \text{ is real, } Z_L \text{ must be as well. Thus, } \lambda/4 \text{ line can only} \\ \text{ - match non-complex (resistive) loads.} \end{array} \right.$

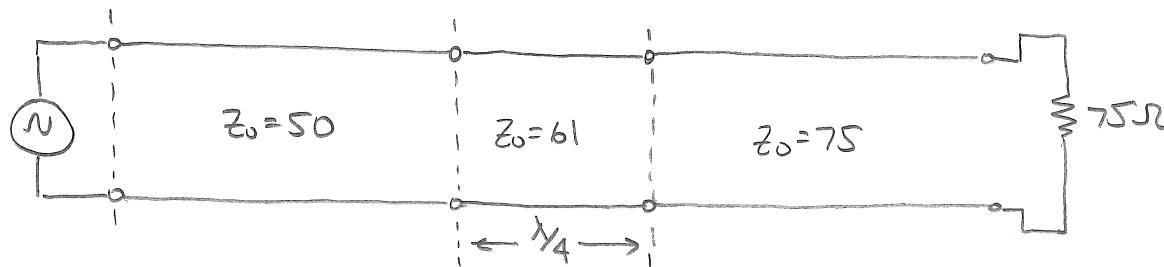
Impedance Transformation ($\frac{\lambda}{4}$ matching)

Match a 100Ω T-Line to a 30Ω resistive load with a $\frac{\lambda}{4}$ matching section.



$$Z_m \text{ will be equal to } \sqrt{Z_L \cdot Z_0}, \text{ so } Z_m = \sqrt{30 \cdot 100} \\ = \underline{\underline{54.8 \Omega}}$$

Since the quarter wave matching works with resistive loads, we should be able to match lines of different Z_0 . For example:



And yes, 62Ω coax does exist! I wonder why?

Feb 23, 15 18:41

quarter_match.sp

Page 1/1

Matching T line to 30 ohms with quarter wave matching network

```
*transmitter with 100 ohm output at 29.6Mhz
Vin vin 0 ac 1.0 sin(0 1.0 29.6e6)
rsrc vin t1_in 100
```

```
*arbitrarily long T-line (about 21 meters)
t1 t1_in gnd t1_out gnd z0=100 F=29.6Meg NL=3.0
```

```
*quarter wave matching coax section
t2 t1_out gnd t2_out gnd z0=54.8 F=29.6Meg NL=.25
```

```
*load is completely resistive, 30 ohms
rload t2_out 0 30
```

```
.control
set hcopydevtype=postscript
* set hcopydev=kec3112-clr
set hcopypscolor=true
set color0=rgb:f/f/f
set color1=rgb:0/0/0
ac lin 100 10M 50Meg
```

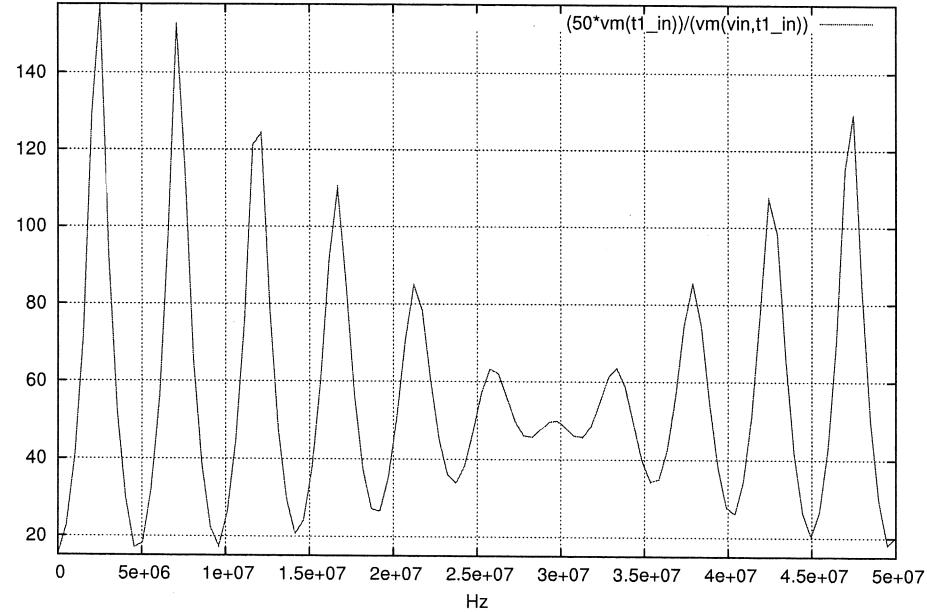
```
*plot I and V at input to T-line
plot vm(vin,t1_in) vm(t1_in)
```

```
*plot T-line input impedance directly
plot (50*vm(t1_in))/(vm(vin,t1_in))
```

```
gnuplot zin.ps (50*vm(t1_in))/(vm(vin,t1_in))
```

```
*hardcopy temp.ps vm(vin) vm(vin,t1_in) vm(join_in)
.endc
.end
```

matching t line to 30 ohms with quarter wave matching network



Feb 23, 15 18:44

quarter_2coax_match.sp

Page 1/1

```

Match 2 different Zo T-lines with 1/4 wave matching

*load is 75 ohms
*feedline to quarter wave matching section is 50 ohms
  50
*transmitter with 100 ohm output at 29.6Mhz
Vin vin 0 ac 1.0 sin(0 1.0 29.6e6)
rsrc vin t1_in 50

*arbitrarily long T-line (about 21 meters)
t1 t1_in gnd t1_out gnd z0=50 F=29.6Meg NL=3.0

*quarter wave matching coax section
t2 t1_in gnd t2_out gnd z0=61.24 F=29.6Meg NL=.25

*arbitrarily long T-line (about 21 meters)
t1 t2_out gnd t3_out gnd z0=75 F=29.6Meg NL=3.0

*load is completely resistive
rload t3_out 0 75

.control
set hcopydevtype=postscript
* set hcopydev=kec3112-clr
set hcopypscolor=true
set color0=rgb:f/f/f
set color1=rgb:0/0/0
ac lin 100 10M 50Meg

*plot I and V at input to T-line
plot vm(vin,t1_in) vm(t1_in)

*plot T-line input impedance directly
plot (50*vm(t1_in))/(vm(vin,t1_in))

gnuplot zin2 (50*vm(t1_in))/(vm(vin,t1_in))

*hardcopy temp.ps  vm(vin) vm(vin,t1_in)  vm(join_in)
.endc
.end

```

