

## Lossy T-lines ; Velocity & Dispersion

Let's think now about the velocity of propagation in the lossy line.

From before, we know  $\beta = \frac{2\pi}{\lambda} = \frac{\omega}{v_p}$ , so { steps:  $\lambda = \frac{v_p}{f}$  &  $\beta = \frac{2\pi}{\lambda}$ ; so by substitution;  $\beta = \frac{2\pi f}{v_p}$   
 $= \frac{\omega}{v_p}$  }  $\Rightarrow v_p = \frac{\omega}{\beta}$   
 $v_p = \frac{\omega}{\beta}$ ; but for a lossy line,  $\beta = \text{Im}\left\{\sqrt{(R+j\omega L)(G+j\omega C)}\right\}$ ; remembering that:

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} \\ = \alpha + j\beta$$

then;  $v_p = \frac{\omega}{\text{Im}\left\{\sqrt{(R+j\omega L)(G+j\omega C)}\right\}}$

then  $v_p$  is dependent on frequency...  
 is this a problem? Yes! But a solution to the problem lurks!

Signals that carry any information must occupy some bandwidth. FM broadcast (stereo) occupies a bandwidth of about 100khz. AM broadcast signals are about 6khz in width. These are analog modulation schemes. What about digital signals?

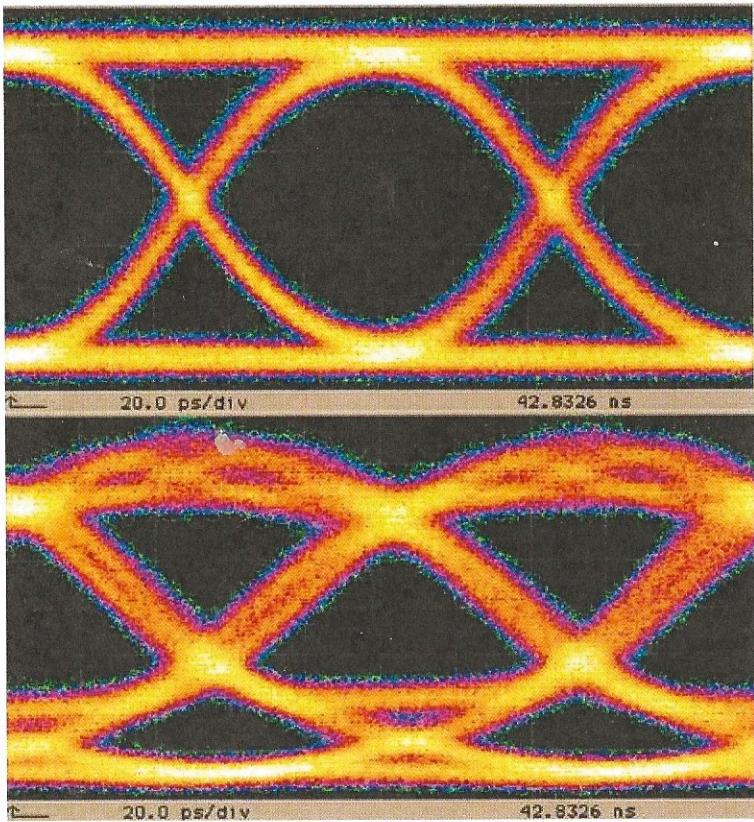
A high-speed digital serial channel has  $t_r+t_f$  on the order of 10's of ps. Many high frequency components make up the edges. What if each frequency travels at a different velocity?

Note: Dispersion is a form of distortion. The waveform shape is changed.

Attenuation is not considered a form of distortion. Attenuation changes amplitude but not shape.

## Lossy T-lines ; Velocity + Dispersion

When each frequency making up an edge travels at a different speed, the waveform edges get "smear" over time and space. Dispersion!



Transmitter signal @  $\approx 5\text{ GHz}$  A very high-speed digital signal.

Received signal after traveling down a lossy transmission line.  
Created with a pseudo random bit stream of 100's of cycles,

(electronics.stackexchange.com)

Lossy lines can strongly effect high-speed digital signals because the different frequencies within the edges travel at different speeds, "smearing" the edge over time.  
What about lower frequencies / longer lines?

## Lossy T-lines; Velocity & Dispersion

The first transatlantic telegraph cables had problems with dispersion. Data rates (Morse code) were very low but the loss was great.

The first message (99 words) from Queen Victoria to President Buchanan took 16 hours to transmit.

Dispersion causes an overlapping between adjacent dots and dashes making them indistinguishable. This is called intersymbol interference (ISI)

Oliver Heaviside (1850-1925) determined that if the parameters for a transmission line were such that  $\frac{R}{L} = \frac{G}{C}$ , the line would exhibit a velocity of propagation that was independent of frequency.

lets see how this works...

"the Heaviside condition"

## Lossy T-lines ; Velocity + Dispersion

From before, we know,

$$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)}$$

lets rearrange this equation so that we can do a substitution easily;

$$\gamma = \sqrt{LC} \left( \frac{R}{L} + j\omega \right) \left( \frac{G}{C} + j\omega \right) ; \text{ now substitute } \frac{R}{L} \text{ for } \frac{G}{C}$$

$$= \sqrt{LC} \left( \frac{R}{L} + j\omega \right) \left( \frac{R}{L} + j\omega \right)$$

$$= \left( \frac{R}{L} + j\omega \right) \sqrt{LC} \quad \begin{matrix} \text{Attenuation} \\ \text{Imaginary part gives us } \beta \text{ (phase constant)} \end{matrix}$$

$$\gamma = \left( \frac{R}{L} \sqrt{LC} \right) + j[\omega \sqrt{LC}]$$

$$\text{so, } \beta = \omega \sqrt{LC}$$

$$\text{thus if } \sqrt{\rho} = \frac{\omega}{\beta} \text{ then } V_p = \frac{\omega}{\omega \sqrt{LC}} = \underline{\frac{1}{\sqrt{LC}}} \quad \begin{matrix} \text{Identical result for } V_p \text{ from} \\ \text{lossless line and is independent} \\ \text{of frequency. Thank goodness} \\ \text{for Heaviside!} \end{matrix}$$

## - Lossy T-lines ; Velocity & Dispersion

For typical T-lines, we find that  $\frac{G}{C} \ll \frac{R}{L}$

- modern dielectrics are really good; thus "G" is very small ( $\approx 10^{-4} \text{ S}$ )
- R is usually copper, the next to best conductor.... which is?...

What to do?

- decrease R?, increase G?, decrease C?, increase L?
- drawbacks, physical considerations?

Solutions:

- increased conductor spacing ( $L$  goes up, and  $C$  goes down)
- fill insulator with iron dust ( $L$  goes up,  $G$  probably goes up)
- distribute inductors along the line
- magnetic coating on conductor (nickel, for example)
- repeaters

See: [wikipedia.org/wiki/Leakage\\_condition](https://en.wikipedia.org/wiki/Leakage_condition)

## Lossy Lines: Fun Facts

Theoretical coaxial cable theory dates back to Civil War times (1800's)

First transatlantic cable had center conductor surrounded by a dielectric made from a latex material derived from the Malaysian gutta percha tree, due to its ability to withstand the intense cold and pressures at the bottom of the ocean.

Seawater was the return path conductor!

The bandwidth of the earliest transatlantic cables was about 1-1.5 Hz!  
By the early 1920's this had improved to about 100 Hz.

Modern coax cable was invented at AT&T. Patent filed May 1929.

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