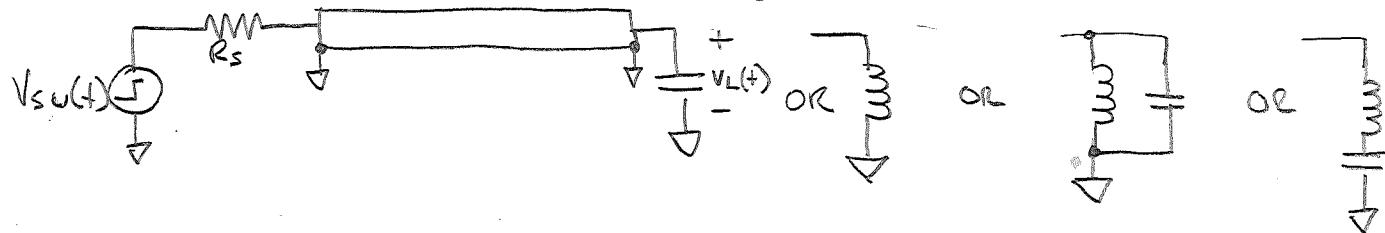


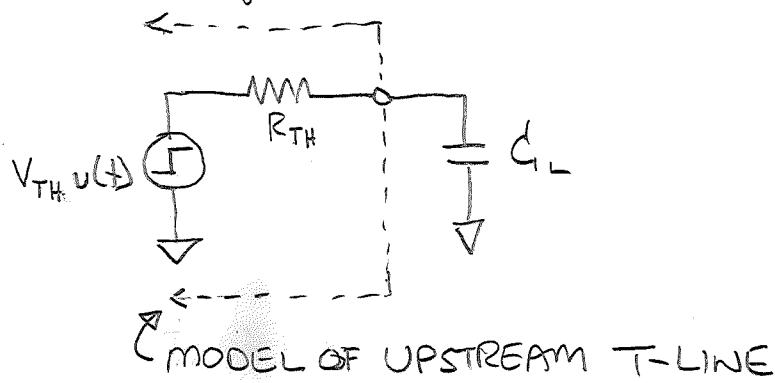
T-lines with Reactive Terminations

What if we have the following situations:



We would like to know what $V_L(t)$ looks like.

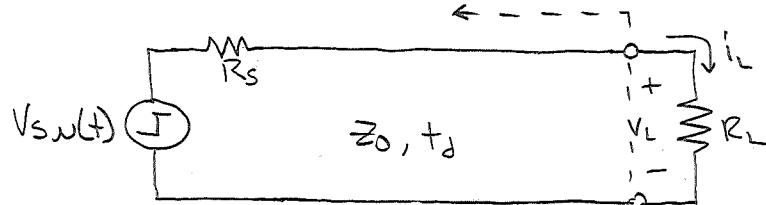
An equivalent model of the T-line would be very helpful. For example:



Let's develop a Thevenin equivalent circuit of the "upstream" transmission line

T-Lines with Reactive Terminations

Here is our T-line to model:



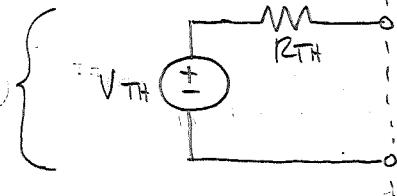
$$V_L(t) = V_1^+ + V_1^-$$

$$= V_{SN}(t - t_d) \left(\frac{Z_0}{R_S + Z_0} \right) + V_{SN}(t - t_d) \left(\frac{Z_0}{R_S + Z_0} \right) P_L$$

$$= V_{SN}(t - t_d) \left(\frac{Z_0}{R_S + Z_0} \right) (1 + P_L)$$

Thevenin model on this side?

Model of the upstream T-line



First, let's find V_{TH} as the open circuit voltage at
At end of an open-circuited T-line. ($R_L = \infty$)

If, at the far end of the line we allow it to be open circuit, $P_L = 1$
Rewriting the equation above,

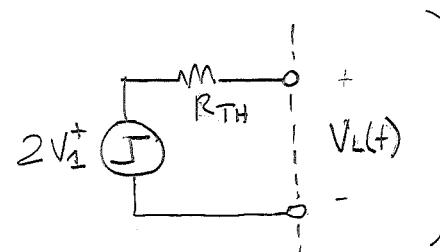
$$V_L(t) = V_{SN}(t - t_d) \left(\frac{Z_0}{R_S + Z_0} \right) (1 + P_L)^{-1}$$

$$= 2V_{SN}(t - t_d) \left(\frac{Z_0}{R_S + Z_0} \right)$$

$$= 2V_1^+ \text{ (twice the incident wave)}$$

$$\text{So } V_T = 2V_1^+$$

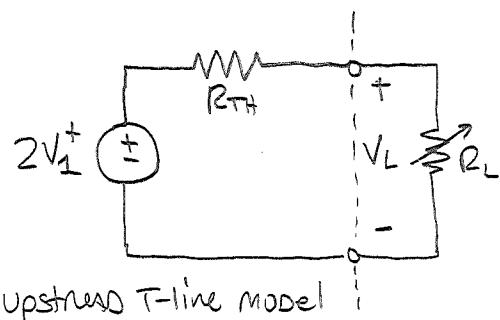
for $t > t_d$



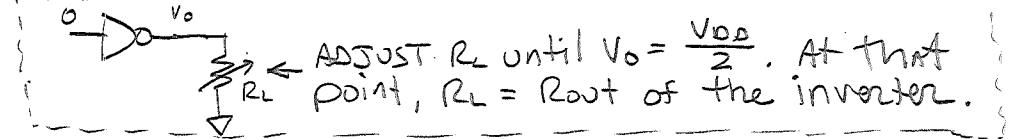
this result agrees with what we know
about T-lines with terminations of $Z_L = \infty$

T-Lines with Reactive Terminations

To determine R_{TH} consider the following circuit:



} This exact empirical method can be used to determine R_{out} of a digital inverter.

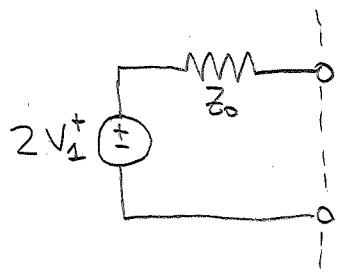


IF we ADJUST R_L until $V_L = \frac{1}{2} 2V_1^+$, then $R_L = R_{TH}$.

Thinking about the T-line Again, what termination applied to the end of the T-line results in $V_L(t) = \frac{1}{2} 2V_1^+ = V_1^+$; in other words, a R_L that results in no reflection?

\rightarrow It's when $R_L = Z_0$! $\therefore R_{TH} = Z_0$

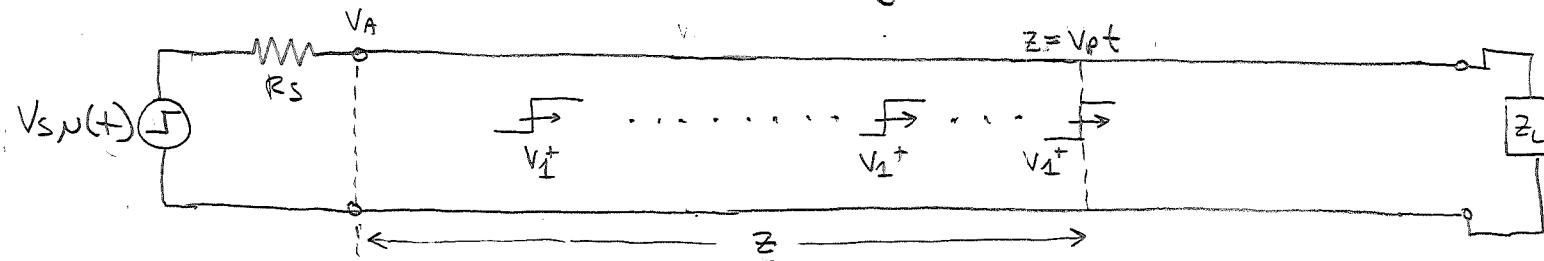
So the model for our upstream T-Line is:



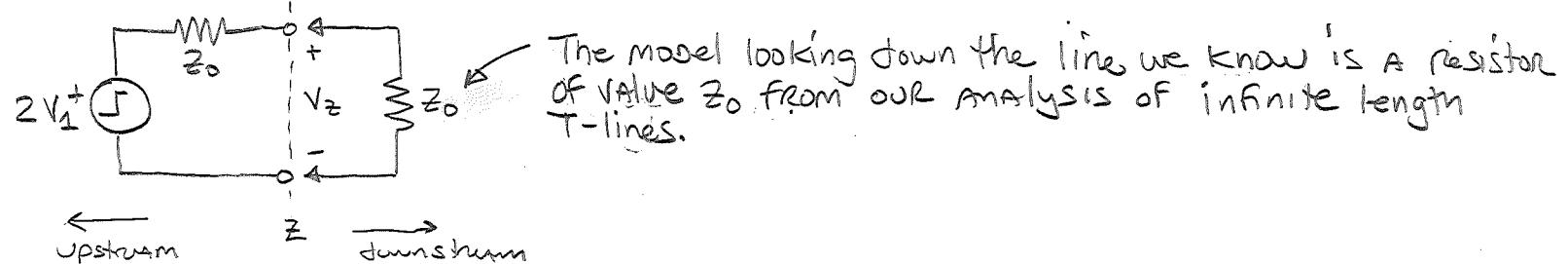
} this is a model that can be universally applied

T-Lines with Reactive Terminations

Our new model also makes sense anywhere upstream on the T-Line.



In the middle of the T-line at z , the model looking upstream is:

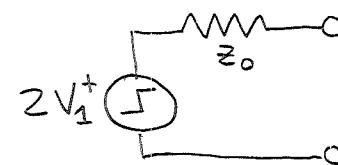


Thus V_z , being the midpoint of a voltage divider will have a wavefront of V_1^+ pass through it only if $V_{TH} = 2V_1^+$ and $R_{TH} = Z_0$, as

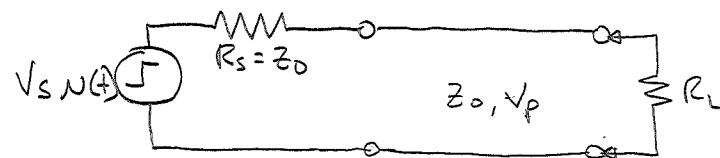
$$V_{z=0} = 2V_1^+ \left(\frac{Z_0}{Z_0 + Z_0} \right) = V_1^+$$

T-lines with Reactive Terminations

The model we have arrived with is:

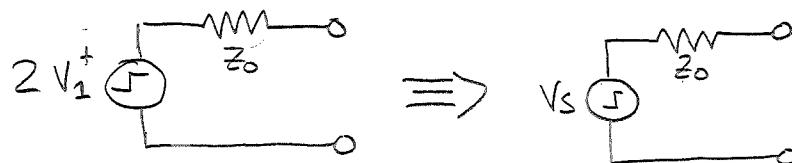


However, we often make a simplifying assumption about our transmission line circuits. We will often place a terminating resistor that matches the T-line so that $P_{\text{source}} = 0$. For example,



R_s allows us to not worry about reflections from the source end.

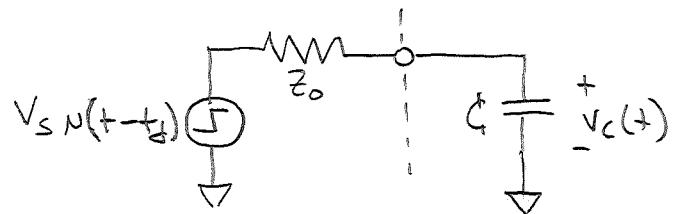
When this is the case, we can take our model & modify it so that it may be transformed as follows:



This is true however, only when $R_s = Z_0$ at the source end of the T-line.

T-Lines with Reactive Terminations

Now, using our model we can redraw the circuit:



Here, t_d represents the delay for the edge to reach the capacitor at the far end (flight time)

This is a familiar circuit and its solution is:

$$V_c(t) = \left[V_c(\infty) + [V_c(t_d^+) - V_c(\infty)] e^{-\frac{(t-t_d)}{\tau}} \right] u(t-t_d)$$

With the capacitor at the end, $V_c(\infty)$ must be V_s . (CAPS are open circuit @ DC)

Also At t_d^+ , when the wave has just arrived, $V_c = 0$. (Voltage Across CAP cannot change instantaneously)
so

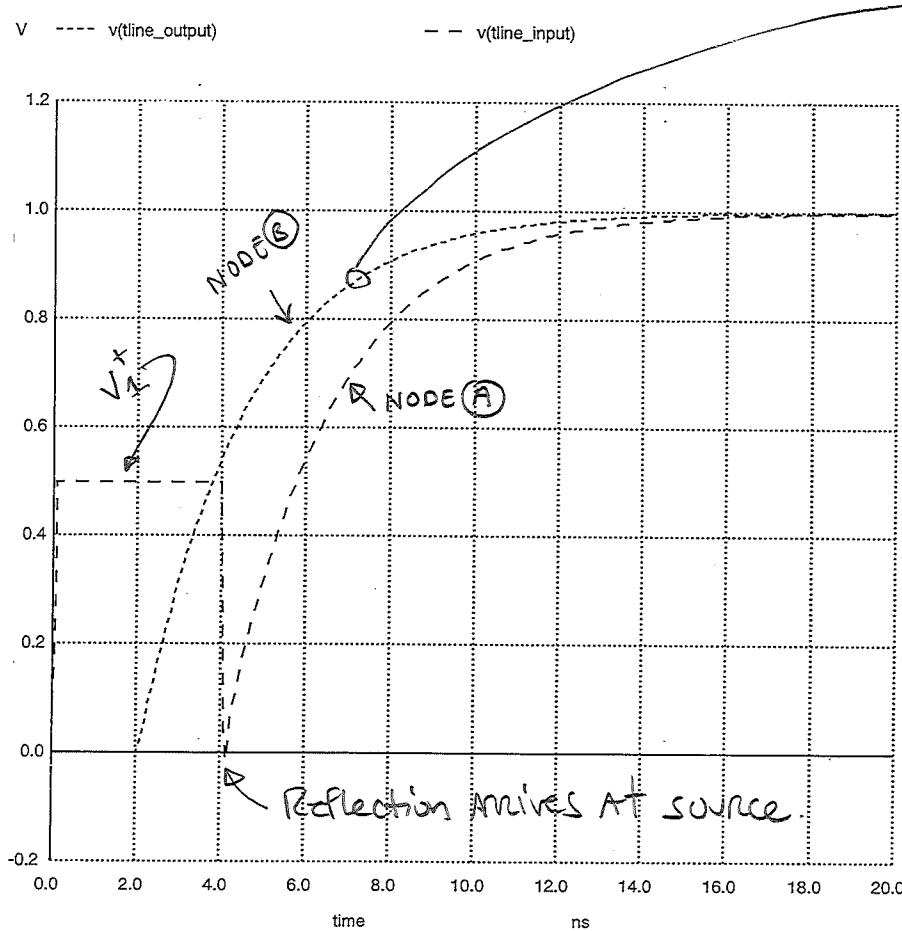
$$\begin{aligned} V_c(t) &= \left[V_s + [0 - V_s] e^{-\frac{(t-t_d)}{\tau}} \right] u(t-t_d) \\ &= \left[V_s - V_s e^{-\frac{(t-t_d)}{\tau}} \right] u(t-t_d) \\ &= V_s \left(1 - e^{-\frac{(t-t_d)}{\tau}} \right) u(t-t_d) \end{aligned}$$

} this is the total voltage at the capacitor formed by both V_i^+ and V_i^-

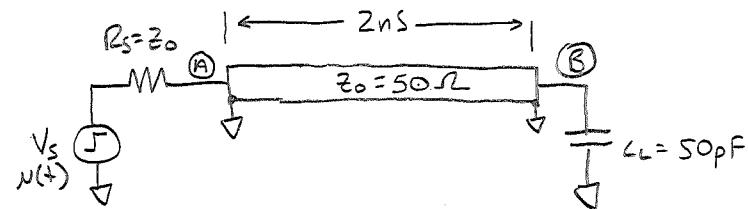
This is the familiar RC charging exponential curve, but delayed by the flight time.

T-Lines with Resistive Terminations

7



This is a total response at the T-Line end. $V_2^+ + V_1^-$. At 2ns, a negative 0.5V value was present for V_1^- , else we would not have 0v.



Apr 17, 15 8:29

cap_term.sp

```

T-line terminated in capacitor or inductor
*input source: delay=0ns, 100ps edges, width=25ns, cycle=50ns
Vin vin 0 1.0 PULSE(0 1.0 0 100p 100p 25e-9 50e-9)

rsrc vin tline_input 50 ;source resistor
t1 tline_input 0 tline_output 0 z0=50 td=2ns ; the t-line

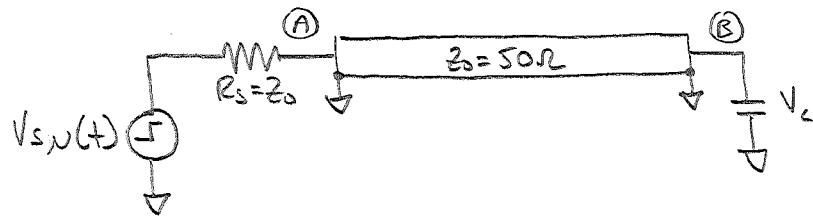
*cload tline_output 0 50p ;terminate line with cap
*lload tline_output 0 250n ;or inductor

.control
  op
  tran 100ps 20ns
  plot v(tline_input) v(tline_output) xl 0.1ns 25ns
.endc
.end

```

Now, let's find the solution at node (A)

T-Lines with Reactive Terminations



AT NODE (A) WE HAVE A TOTAL VOLTAGE VOLTAGE AFTER $2t_d$ OF:

$$V_A(t) = V_1^+ + V_1^- \quad \text{we need to know } V_1^-; \text{ but we know } V_1^+ + V_1^- + V_1^- \text{ at the far end.}$$

At (B) from before, $V_c(t) = V_s(1 - e^{-\frac{(t-t_d)}{\tau}}) u(t-t_d)$, since this is a total voltage,

$$\text{e(B)} \quad V_1^+ + V_1^- = V_s(1 - e^{-\frac{(t-t_d)}{\tau}}) u(t-t_d), \text{ and we know } V_1^+ \text{ as } V_s \left(\frac{Z_0}{Z_0 + R_s} \right)$$

$$\text{e(B)} \quad V_1^- = \left[V_s \left(1 - e^{-\frac{(t-t_d)}{\tau}} \right) - V_1^+ \right] u(t-t_d) \quad \left. \begin{array}{l} \text{At } t=t_d \text{ a reflection of } V_1^+ \\ -V_1^+ \text{ is generated. This gives} \\ \text{us total voltage of zero at} \\ \text{the end point} \end{array} \right\}$$

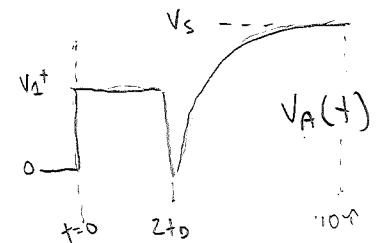
Then at (A), which is another t_d away in time,

$$\begin{aligned} V_A(t) &= V_1^+ + V_1^- \\ &= V_1^+ u(t) + \left[V_s \left(1 - e^{-\frac{(t-2t_d)}{\tau}} \right) - V_1^+ \right] u(t-2t_d) \quad \left. \begin{array}{l} \text{incident wave} \\ \text{reflection delayed by } 2t_d \end{array} \right\} \text{for all } t \\ &= V_1^+ + V_s \left(1 - e^{-\frac{(t-2t_d)}{\tau}} \right) - V_1^+ \quad \left. \begin{array}{l} \text{for } t > 2t_d \end{array} \right\} \end{aligned}$$

$$V_A(t) = V_s \left(1 - e^{-\frac{(t-2t_d)}{\tau}} \right) \text{ for } t > 2t_d \quad || \text{ exactly what we saw at (B), just delayed by } t_d$$

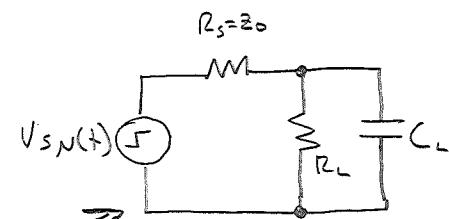
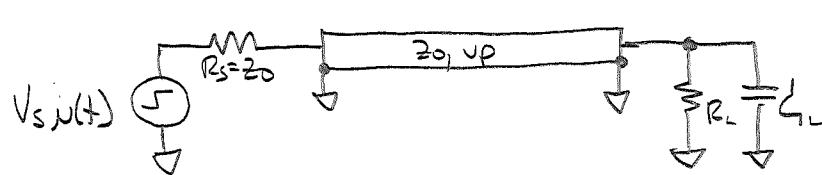
Which gives us the correctly shaped curve at (A)

$$\text{BTW, } \tau = R\ell$$



T-Lines with Reactive Terminations

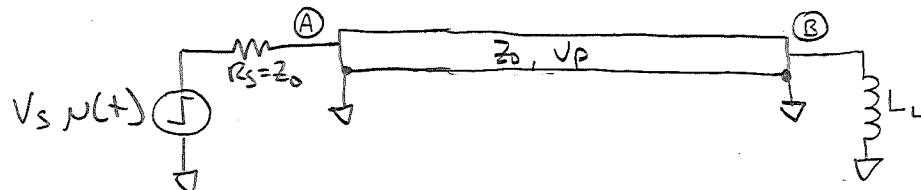
What about:



Use a Thevenin equivalent ckt. What would it be?

$\Upsilon = ?$

How about this?



Remember the general form for 1st order RL, RC circuits

$$x(t) = x(\infty) + [x(t_0^+) - x(\infty)] e^{-\frac{(t-t_0)}{\Upsilon}} \quad ; \text{ where } \Upsilon = \frac{L}{R}$$

final value initial value final value
 $\frac{(t - \text{time switching occurs})}{\text{time constant}}$

Remember:

Inductors are an open circuit to high-frequency currents.

Inductors are a short circuit to DC currents

At the loss end of our T-Line its:

$$V_L(t) = V_L(\infty) + [V_L(t-t_0) - V_L(\infty)] e^{-\frac{(t-t_0)}{\Upsilon}} (u(t-t_0))$$

T-Lines with Reactive Terminations

10

At time = 0, the voltage across the inductor is zero

At time = t_d^+ , $V_L(t)$ will be V_s . As the voltage is allowed to change so at the load end, instantaneously ($V_s = 2V_1^+$)

$$V_L(t) = [0 + [V_s - 0] e^{-\frac{(t-t_d)}{\tau}}] u(t-t_d)$$

$$= \left[V_s e^{-\frac{(t-t_d)}{\tau}} \right] u(t-t_d) \quad \} \text{ this is a total voltage, i.e. } V_1^+ + V_1^- \text{, so}$$

$$V_1^- = \left[V_s e^{-\frac{(t-t_d)}{\tau}} - V_1^+ \right] u(t-t_d) \quad \} \text{ this is just the reflected wave launched from the load end. (At } t=t_d^+ \text{ it's } \frac{1}{2}V_s \text{ or } V_1^+ \text{)}$$

AT THE SOURCE END (A) :

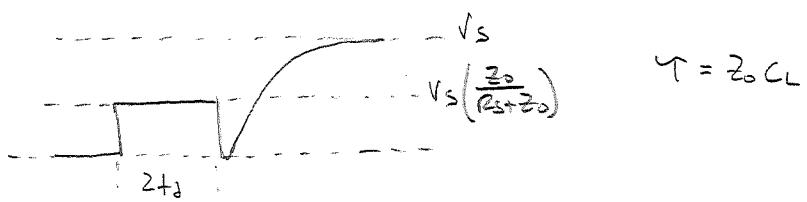
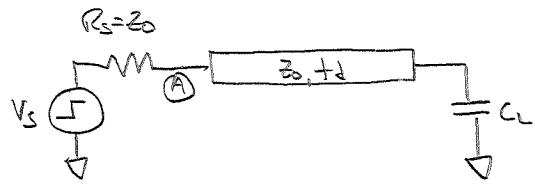
$$V_A(t) = V_1^+ + V_1^- = V_1^+ u(t) + \underbrace{\left[V_s e^{-\frac{(t-2t_d)}{\tau}} - V_1^+ \right] u(t-2t_d)}_{\text{reflected wave}} \quad \Rightarrow \quad \begin{array}{c} (\text{Reflection arrives after } 2t_d) \\ \text{Graph: } V_A \text{ vs } t \end{array}$$

so at (A) for $t > 2t_d$

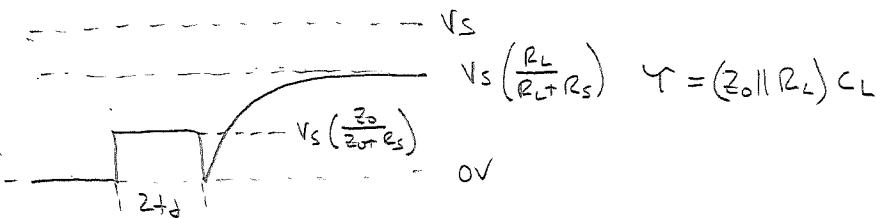
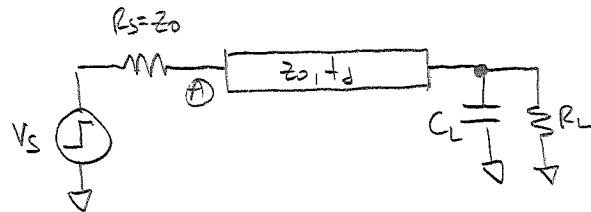
$$V_A(t) = V_1^+ + V_s e^{-\frac{(t-2t_d)}{\tau}} - V_1^+$$

$$V_A(t) = V_s e^{-\frac{(t-2t_d)}{\tau}} \quad (t > 2t_d)$$

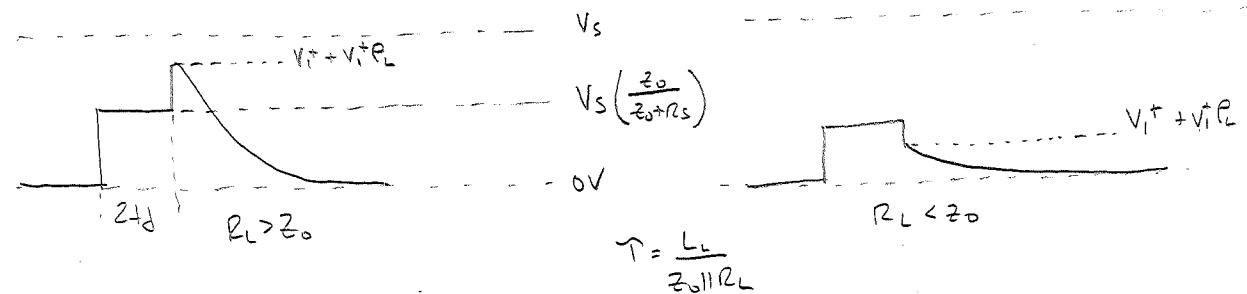
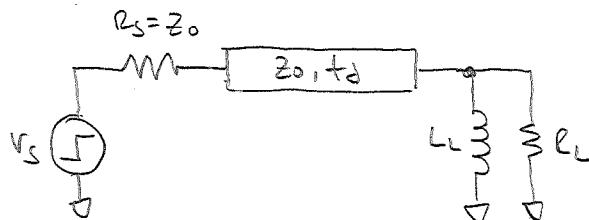
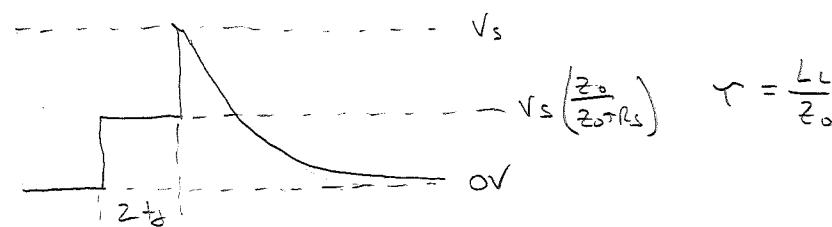
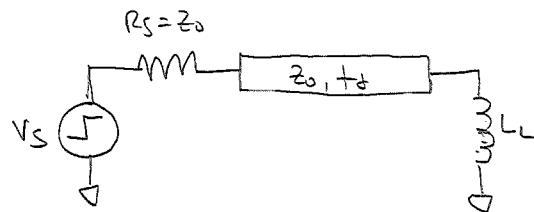
which is what we would expect to see.



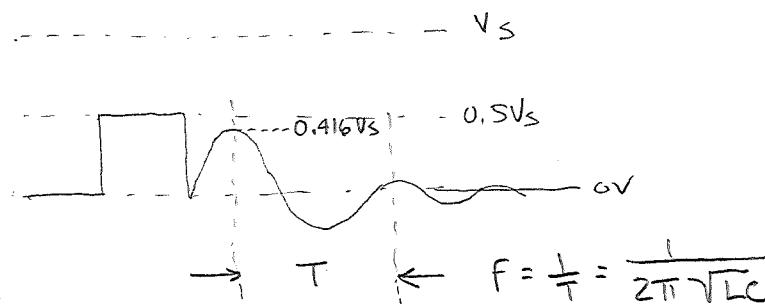
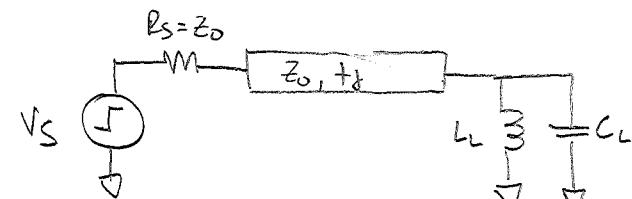
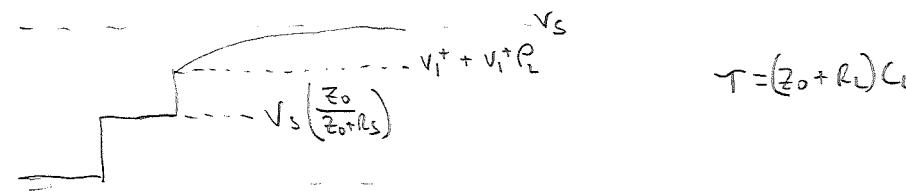
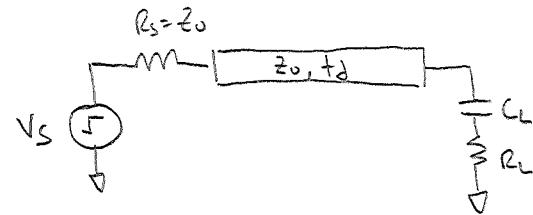
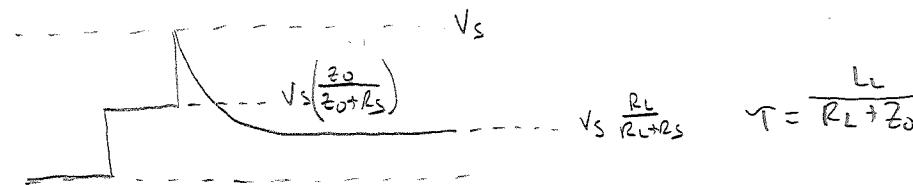
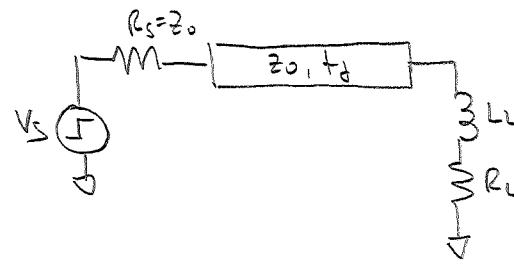
Use : reactive-terms sp



SUMMARY OF
reactive termination
waveforms AT input
to T-line (pt A)



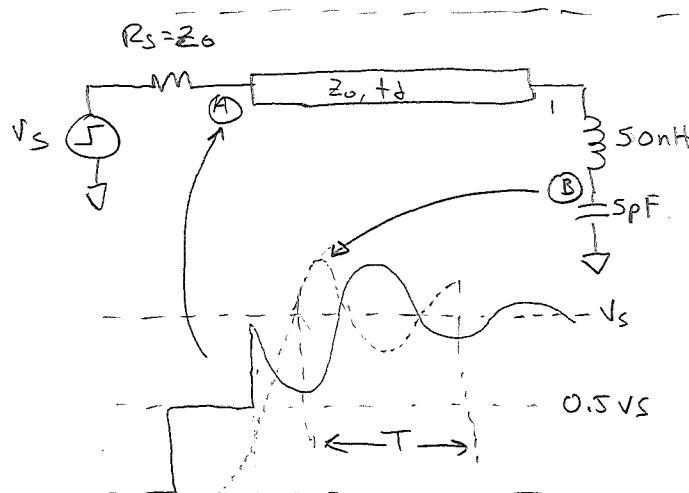
Summary of Reactive Termination Waveforms
At input to T-line (pt A)



for $L_L = 100\text{nH}$,
 $C_L = 100\text{pF}$

$T = 21\text{ns}$ measured on spice plot
 $f = 47.4\text{MHz}$

$\frac{1}{2\pi\sqrt{LC}} = 50.35\text{MHz}$, close correlation



$$\frac{1}{T} = \frac{1}{2\pi\sqrt{LC}}$$