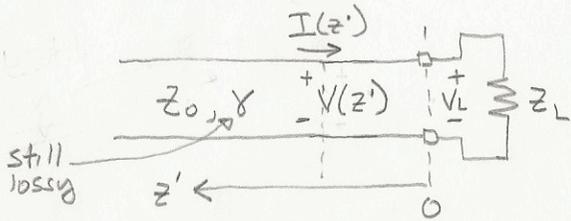


Reflections on Terminated Lines (steady state)

A terminated, lossy T-line:



- here we introduce a different point of reference for measuring distance. From now on we measure a distance z' ; the physical distance from the load.
- In the RF world, we are delivering power to a remote load. We are interested in what is happening there.

So, now we have:

$$V(z') = V_0^+ e^{\gamma z'} + V_0^- e^{-\gamma z'} \quad \text{And,} \quad I(z') = \frac{V_0^+}{Z_0} e^{\gamma z'} - \frac{V_0^-}{Z_0} e^{-\gamma z'}$$

{ only change is with the signs of the exponents as the point of reference has changed. }

Suppose we want to find Z_L using these two equations. At the load ($z'=0$) we have:

$$V(z'=0) = V_0^+ e^{\gamma(0)} + V_0^- e^{-\gamma(0)}$$

At load \uparrow

$$= V_0^+ + V_0^-$$

V_0^+ is not quite like V_1^+ of the transient case, since we are in S.S. AC, the source is continually launching a wave, so, V_0^+ is the voltage at the generator side of the T-line.

likewise,

$$I(z'=0) = \frac{V_0^+}{Z_0} e^{\gamma(0)} - \frac{V_0^-}{Z_0} e^{-\gamma(0)}$$

At load \uparrow

$$= \frac{V_0^+ - V_0^-}{Z_0}$$

Now, knowing the current (I_L) and voltage (V_L) at the load using Ohm's law, we get:

$$Z_L = \frac{V_L}{I_L} = \frac{V_0^+ + V_0^-}{\frac{V_0^+ - V_0^-}{Z_0}}$$

$$Z_L = Z_0 \left(\frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \right)$$

this equation relates:

- Z_0 of the line
- the load impedance
- forward + reflected voltages

A very useful equation. there exist ways to independently measure $V_0^- + V_0^+$; An SWR meter we will come back to this.

Reflections on Terminated Lines (Steady State)

What we would like to know is the amount of reflected voltage and/or current we have from a load $(\frac{V_o^-}{V_o^+})$ as a function of the load impedance and the line Z_o . In most cases, this is a quantity we wish to minimize to maximize power delivered to a load.

From before, we had, $Z_L = Z_o \left(\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} \right)$ { this eqn has what we need to find $\frac{V_o^-}{V_o^+}$ as a function of $Z_o + Z_L$.

Solve for V_o^- as a function of V_o^+ :

$$Z_L = Z_o \left(\frac{V_o^+ + V_o^-}{V_o^+ - V_o^-} \right)$$

$$Z_L (V_o^+ - V_o^-) = Z_o (V_o^+ + V_o^-)$$

$$Z_L V_o^+ - Z_L V_o^- = Z_o V_o^+ + Z_o V_o^-$$

$$-Z_o V_o^- - Z_L V_o^- = Z_o V_o^+ - Z_L V_o^+$$

$$-V_o^- (-Z_L - Z_o) = V_o^+ (Z_o - Z_L) \Rightarrow \frac{V_o^-}{V_o^+} = \frac{Z_o - Z_L}{-Z_L - Z_o} = \frac{Z_L - Z_o}{Z_L + Z_o}$$

$$\frac{V_o^-}{V_o^+} = \frac{Z_L - Z_o}{Z_L + Z_o} = \underline{\underline{\Gamma_L}}$$

{ the load voltage reflection coefficient (gamma) } (like P_L from before)

(voltage) (current)

We can also derive the current reflection coefficient where $\Gamma_V = -\Gamma_I$

Note that Γ_L is complex, thus,

$$\Gamma_L = |\Gamma_L| e^{j\theta_L} \quad -1 \leq \Gamma_L \leq 1$$

↑ mag ↑ phase rotation $|\Gamma_L| \leq 1$

$$\Gamma_L = \frac{V_o^-}{V_o^+} \text{ (definition of } \Gamma_L)$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} \quad Z_L = Z_o \left(\frac{1 + \Gamma_L}{1 - \Gamma_L} \right)$$

$$\Gamma_L = |\Gamma_L| e^{j\theta_L} ; \text{ where } |\Gamma_L| = \sqrt{\text{real}^2 + \text{imag}^2} + \theta_L = \tan^{-1} \left(\frac{\text{imag}}{\text{real}} \right)$$

(phase angle of the reflection is a function of $Z_o + Z_L$) (just like before!)

Reflections on Terminated Lines (Steady State)

Using Γ_L we can rewrite our voltage equation $\hat{V}(z')$ in terms of V_0^+ without V_0^- .

At this point we will also restrict ourselves to lossless lines. Thus,

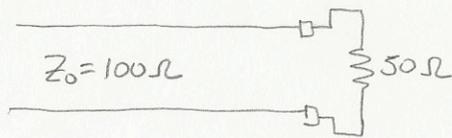
$$V(z') = V_0^+ e^{j\beta z'} + \Gamma_L V_0^+ e^{-j\beta z'} \quad \leftarrow \text{(the reflected wave } V_0^-)$$

And,

$$I(z') = \frac{V_0^+}{Z_0} e^{-j\beta z'} - \Gamma_L \frac{V_0^+}{Z_0} e^{+j\beta z'}$$

↑ otherwise at this point it gets too messy + obscures what we are trying to do.

Let's consider what Γ_L is telling us and also visualize it.

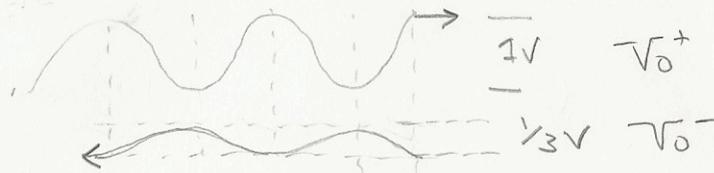


$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 100}{50 + 100} = -\frac{1}{3}$$

$$\Gamma_L = |-\frac{1}{3}| e^{j\pi} \quad \leftarrow \text{OR } \leftarrow$$

resistive load $\rightarrow \Gamma_L = |\Gamma_L| e^{j(0)}$ or $|\Gamma_L| e^{j\pi}$
 \rightarrow the phase rotation due to load is either 0° or 180° with resistive loads.

So if we have a V_0^+ of 1V Amplitude
 We will have a V_0^- of $-\frac{1}{3}$ V Amplitude



The minus sign indicates a phase reversal. (more about this later...)

The two waves will ADD/SUBTRACT AT ALL points on the line.

The result is an envelope of Amplitude with peaks + valleys down the line.

(Watch AT+T video "Similarities of Wave Behavior", 10:33 - 15:30)

Let's look at 3 cases, open, shorted + terminated ($Z_L = Z_0$) lines

Reflections on Terminated Lines (Termination is o.c.)

$$Z_0 = 100\Omega \quad \text{o.c.} \quad \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{\infty - Z_0}{\infty + Z_0} = \underline{\underline{1}}$$

At the load:

$$V(z'=0) = V_0^+ e^{j\beta z'} + \Gamma V_0^+ e^{-j\beta z'} \\ = 2V_0^+ \text{ at the load}$$

IF V_0^+ is +1V, then V_0^- will be +1V
At the load, the voltage is doubled, the sign is positive, no phase inversion at load

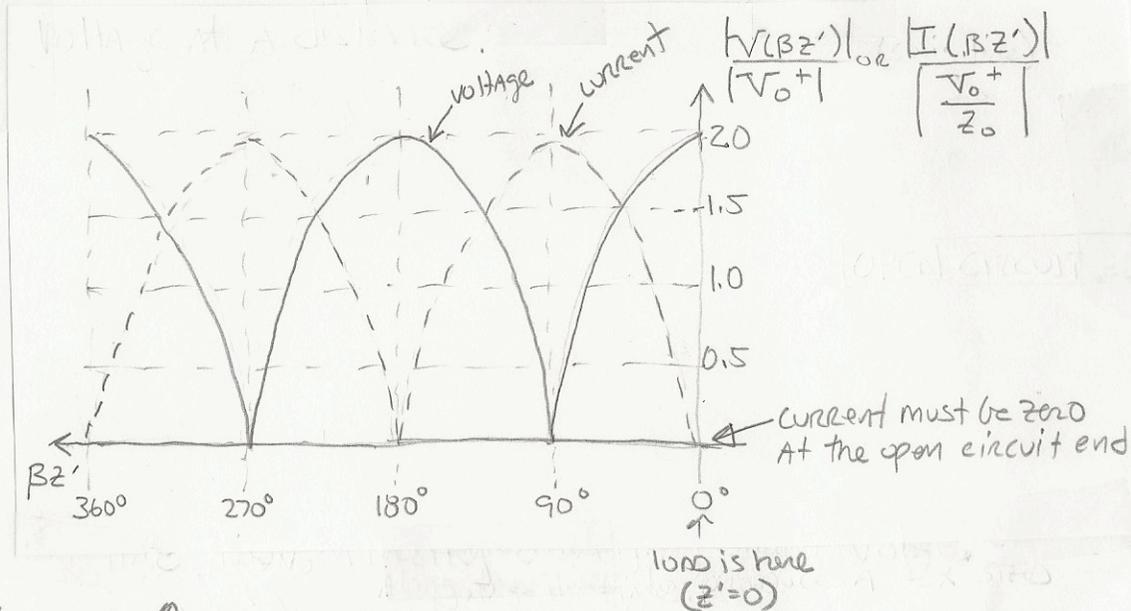
→ Moving away from the load, $V(z') + I(z')$ look like this:

Elsewhere on the line:

$$V(z') = V_0^+ e^{j\beta z'} + \Gamma V_0^+ e^{-j\beta z'} \\ = V_0^+ (e^{j\beta z'} + e^{-j\beta z'}) \quad \left\{ \cos(x) = \frac{e^{ix} + e^{-ix}}{2} \right. \\ = \underline{\underline{2V_0^+ \cos(\beta z')}}$$

Since $I(z') = \frac{1}{Z_0} [V_0^+ e^{j\beta z'} - \Gamma V_0^+ e^{-j\beta z'}]$

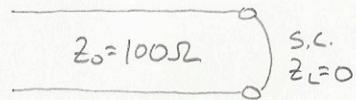
$$I(z') = \frac{V_0^+}{Z_0} [e^{j\beta z'} - e^{-j\beta z'}] \quad \left\{ \sin(x) = \frac{e^{ix} - e^{-ix}}{2i} \right. \\ = \underline{\underline{\frac{2V_0^+}{Z_0} j \sin(\beta z')}}$$



Remember, this is magnitude versus distance
note: Location of V_{min} = location of I_{max}
; This graph is shown as an envelope at RFmentor.com

visit : www.rfmentor.com and play. Try $Z_L = \infty$

Reflections on Terminated Lines (termination is S.C.)



$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \underline{\underline{-1}}$$

- If V_0^+ is 1V, then V_0^- is -1V
- At the load, the voltage must be zero
- Sign of Γ is negative, phase inversion occurs at Z_L

At the load: ($z'=0$)

$$V(z'=0) = V_0^+ e^{j\beta z'} + \Gamma_L V_0^+ e^{-j\beta z'} \\ = V_0^+ + (-1)V_0^+ = \underline{\underline{0V \text{ at the load}}}$$

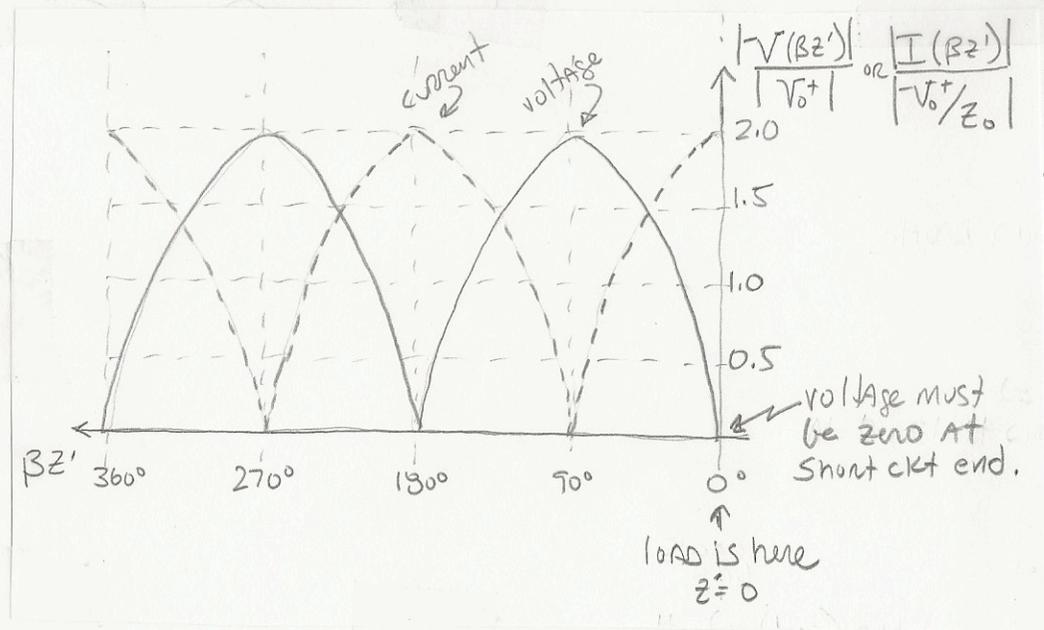
Elsewhere on the line:

$$V(z') = V_0^+ e^{j\beta z'} + \Gamma_L V_0^+ e^{-j\beta z'} \\ = V_0^+ e^{j\beta z'} - V_0^+ e^{-j\beta z'} \quad \text{Shorted line, } V_0^- = -V_0^+ \\ = V_0^+ (e^{j\beta z'} - e^{-j\beta z'}) \quad \parallel \sin(x) = \frac{e^{jx} - e^{-jx}}{2j} \\ = \underline{\underline{2V_0^+ j \sin(\beta z')}}$$

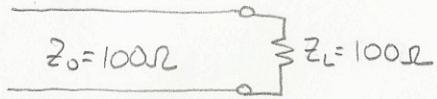
Likewise, $I(z') = \frac{V_0^+ e^{j\beta z'}}{Z_0} - \frac{V_0^- e^{-j\beta z'}}{Z_0}$

$$= \frac{1}{Z_0} [V_0^+ e^{j\beta z'} - \Gamma_L V_0^+ e^{-j\beta z'}] \\ = \frac{V_0^+}{Z_0} [e^{j\beta z'} + e^{-j\beta z'}] \\ = \underline{\underline{\frac{2V_0^+}{Z_0} \cos(\beta z')}}$$

Moving AWAY from the load, $V(z')$ & $I(z')$ look like this:



Reflections on Terminated Lines



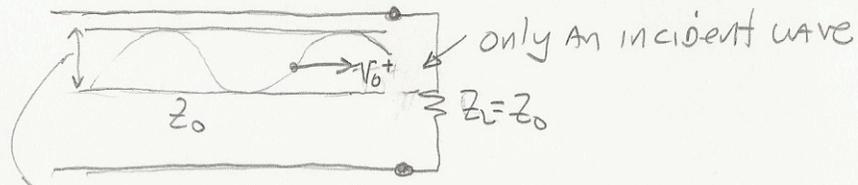
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0}{200} = 0 \quad \therefore \text{There are no reflections, this is a matched line.}$$

At the load: $V(z'=0) = V_0^+ e^{j\beta z'} + \cancel{\Gamma V_0^+ e^{-j\beta z'}} = V_0^+ e^{j\beta z'}$ at the load

Elsewhere on the line:

$$V(z') = V_0^+ e^{j\beta z'} + \cancel{\Gamma V_0^+ e^{-j\beta z'}} = V_0^+ e^{j\beta z'}$$

Identical Result. So, At Any point on the line including the load, we have just the incident wave. Also, the amplitude is identical everywhere.



Amplitude envelope is the same everywhere on the line.