

Analysis in the Steady State (A quick review of some basics and a justification for use of phasors)

Sinusoidal sources produce both a transient response AND a steady state response.

Transient responses die out with time and leave only the steady state response.

When the transient response becomes negligibly small, we say that a system is operating at sinusoidal steady state.

Sinusoids may be conveniently expressed as phasors. A phasor is a complex number that represents the amplitude + phase of a sinusoid. A phasor represents a time-varying sinusoidal waveform by a fixed complex number.

In RF circuits, we are most interested in delivering power to a load, which is often reactive by nature.

Analysis in the Steady State

Phasor representation is based on Euler's Identity

$$e^{+jx} = \cos x + j \sin x \quad e^{-jx} = \cos x - j \sin x \quad \left\{ \begin{array}{l} \text{Also, } e^{i\pi} + 1 = 0 \text{ or } e^{i\pi} = -1 \\ \text{"the most beautiful equation"} \end{array} \right.$$

We can consider $\cos x$ as the real part of e^{jx} ; $\cos x = \operatorname{Re}\{e^{jx}\}$
 And, $\sin x$ as the imaginary part; $\sin x = \operatorname{Im}\{e^{jx}\}$

$\xrightarrow{\text{time domain}}$ $\xrightarrow{\text{magnitude}}$ $\xrightarrow{\text{Arbitrary reference phase}}$

Given we have a signal: $v(t) = V_m \cos(\omega t + \phi)$ then, since $\cos x = \operatorname{Re}\{e^{jx}\}$;

$$\begin{aligned}
 &= \operatorname{Re}\{V_m e^{j(\omega t + \phi)}\} \\
 &= \operatorname{Re}\left\{V_m e^{j\phi} e^{j\omega t}\right\} \\
 &\quad \text{not time varying, thus its} \\
 &\quad \text{A capital letter, or sometimes} \\
 &\quad \text{boldface font. The phasor encompasses} \\
 &\quad \text{the vector length, which does not change.} \\
 \text{this is still a time} & \quad \left. \begin{aligned} &v(t) = \operatorname{Re}\left\{V e^{j\omega t}\right\} : \text{where } V = V_m e^{j\phi} \\ &V \text{ is the phasor representation of } v(t) \end{aligned} \right\} \\
 \text{varying waveform} &
 \end{aligned}$$

The phasor "V" captures the amplitude and phase of a sinusoid but does not provide information about its frequency.

We are assuming a linear circuit where ω does not change and no new frequencies are created. This is a definition of a linear system.

The angular frequency ω is assumed to be known and is the only frequency present.

We are just interested in phase + magnitude, not how fast the vector is rotating (ω = angular velocity)
 So now, we focus on $V = V e^{j\phi}$

Having a phasor in phase + magnitude, it has fixed orientation.

Analysis in the Steady State

For a sinusoidal wave on a transmission line we must keep track of both time & space. From before, (wk1)

$$v(z,t) = \cos\left(2\pi ft - 2\pi \frac{z}{\lambda} + \phi\right); \text{ let } \beta = \frac{2\pi}{\lambda}$$

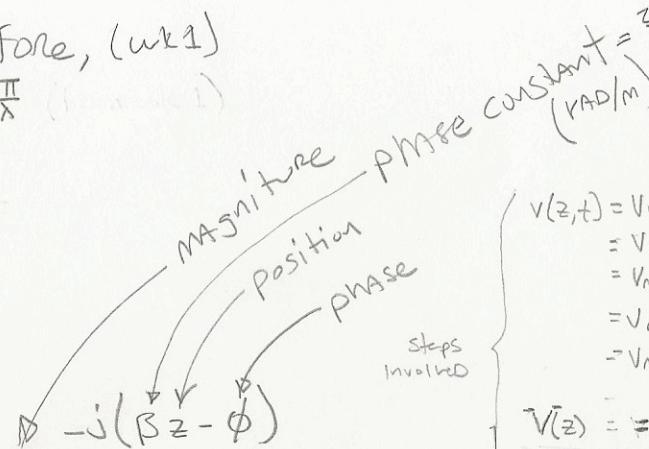
(now, allow scaling by a magnitude V_m)

$$v(z,t) = V_m \cos(\omega t - \beta z + \phi)$$

$$\rightarrow v(z,t) = \operatorname{Re} \left\{ V_m e^{-j(\beta z - \phi)} e^{j\omega t} \right\}$$

A function of time (t) & space (z)

$$\text{And the voltage phasor is: } \tilde{V}(z) = V_m e^{-j(\beta z - \phi)}$$



$$\begin{aligned} v(z,t) &= V_m \cos(\omega t - \beta z + \phi) \\ &= V_m \operatorname{Re} \{ e^{j(\omega t - \beta z + \phi)} \} \\ &= V_m \operatorname{Re} \{ e^{\beta j t} e^{\omega j t} e^{-j \beta z} e^{j \phi} \} \\ &= V_m \operatorname{Re} \{ e^{\beta j t} \cdot e^{-j \beta z} \cdot e^{j \phi} \} \\ &= V_m \operatorname{Re} \{ e^{\beta j t} \cdot e^{-j(\beta z - \phi)} \} \\ &\quad (\text{throw out time varying part}) \\ \tilde{V}(z) &= V_m e^{-j(\beta z - \phi)} \end{aligned}$$

only a function of z, ϕ + not of time

$$\cos \theta = \operatorname{Re} \{ e^{j\theta} \}$$

Apply this rule

remember
 $e^{M+n} = e^M * e^n$
 $e^{M-n} = \frac{e^M}{e^n}$

- We only need concern ourselves with magnitude and phase - for our solutions,
- There is time domain behavior present on a T-Line but it's of relatively no interest.
- DC Circuits had dimensions of only magnitude
- Lumped AC Circuits have dimensions of magnitude + phase (linear, steady state)
- Transmission Lines have dimensions of magnitude, phase + distance.
- We can use phasors alone because of linearity