

# Traveling Waves

## Properties:

- They carry energy and can exert force at a distance
  - We exploit this property to transmit information
- They have velocity
  - Waves take time to travel from one point to another. The velocity they travel at depends on the medium they travel through.
- They exhibit linearity
  - Waves can pass through each other without affecting the passage of other waves. The total of 2 linear waves is simply the sum of the individual wave amplitudes at any point in time or space.

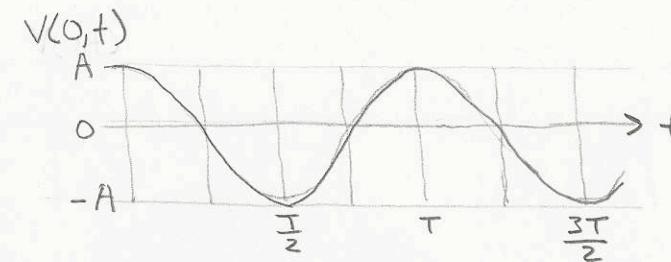
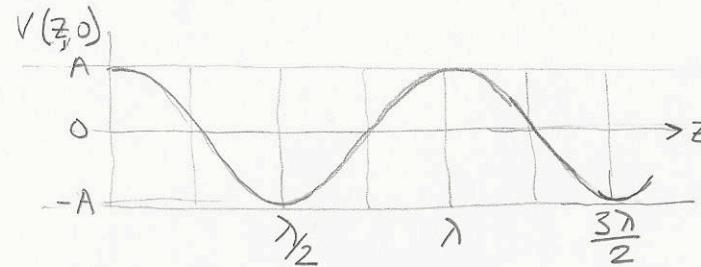
## Mathematical Representation:

- Assuming a lossless medium (Amplitude is not diminished with distance) the voltage of a wave traveling on a transmission line can be expressed as:  $V(z,t) = A \cos(2\pi F t - 2\pi \frac{z}{\lambda} + \phi_0)$ ; where  $A$  = Amplitude of the wave  $F$  = Frequency of the wave  $\lambda$  = Spatial wavelength
- Cosine or sin may be used
- the "phase" of the wave  
degrees or radians
- reference phase

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Let's look at the case where  $\phi_0 = 0$ .

Also, we can rewrite our equation as:  $V(z, t) = A \cos(2\pi \frac{t}{T} - 2\pi \frac{z}{\lambda})$  And observe  $V(z, 0)$  and  $V(0, t)$ :



time is frozen @  $\phi$ , walk down the line

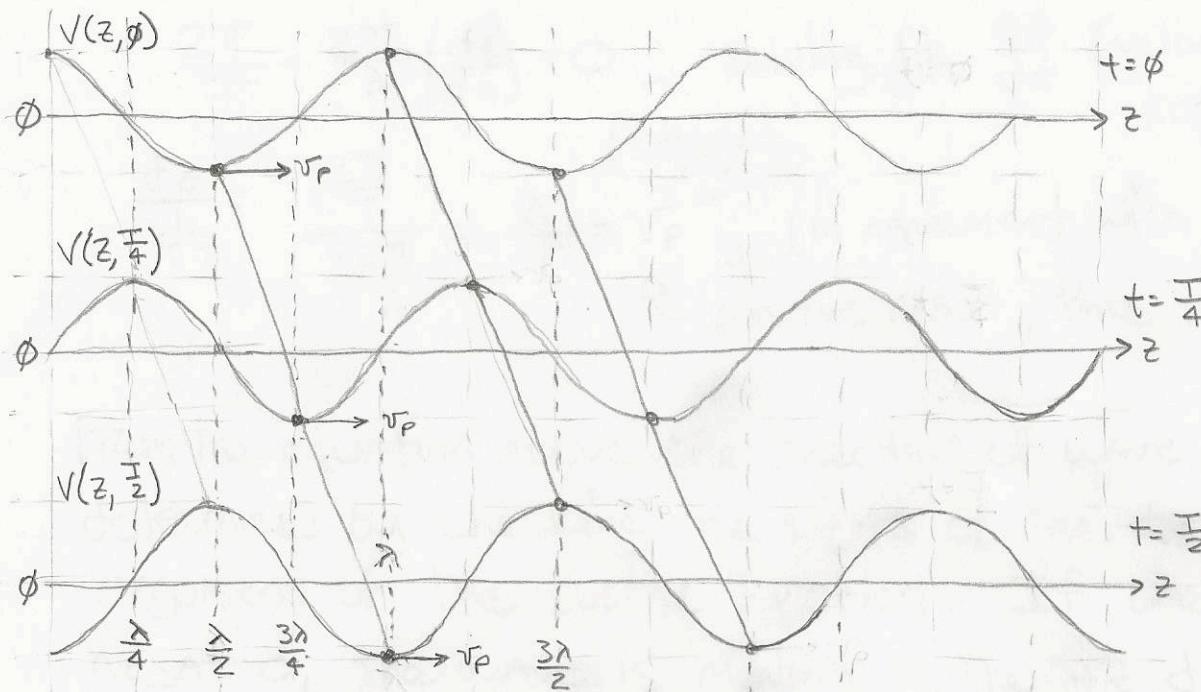
freeze the position, observe voltage over time

The wave repeats at a spatial period of  $\lambda$

" " " " temporal period of  $T$

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Now, let's plot  $v(z, t)$  at 3 different points in time:



by tracking one point in time, we can find  
that  $v_p$  of that point is:

$$v_p = \frac{\Delta}{\frac{T}{2}} = \frac{\Delta}{T} \left( \frac{\text{distance}}{\text{time}} \right)$$

We see that the pattern of wave movement is in the  $+z$  direction as time increases.  
Therefore, we say the wave is traveling in the  $+z$  direction.

If we track a point on the wave we can determine its phase velocity. If we choose a point  $v_0$  to track, we could say:

(Also known as propagation velocity  $v_p$ )

$$v_0 = A \cos \left( 2\pi \frac{t}{T} - 2\pi \frac{z}{\lambda} \right) : \text{then}$$

$$2\pi \frac{t}{T} - 2\pi \frac{z}{\lambda} = \left[ \cos^{-1} \frac{v_0}{A} \right] \quad \text{this is a constant}$$

The velocity of the point  $v_0$  can be found by taking the time derivative of the above equation.

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Taking the derivative gives:

$$\frac{2\pi}{T} - \frac{2\pi}{\lambda} \left( \frac{dz}{dt} \right) = 0 : \text{solving for } \frac{dz}{dt} \text{ (velocity)}$$

$$\frac{dz}{dt} = \frac{-\frac{2\pi}{T}}{\frac{-2\pi}{\lambda}} = \frac{\lambda}{T} = v_p j \quad (\text{in agreement with the drawing})$$

↑ positive result, thus moving in +z direction

From the equation above the direction of wave propagation can be determined by checking the signs of the  $t + z$  terms in the argument of the cosine function. If one is positive and one negative, the wave is moving in the +z direction. If both are positive or negative, it's moving in the -z direction.