

Z'_{\min} and Z'_{\max} of Standing Waves

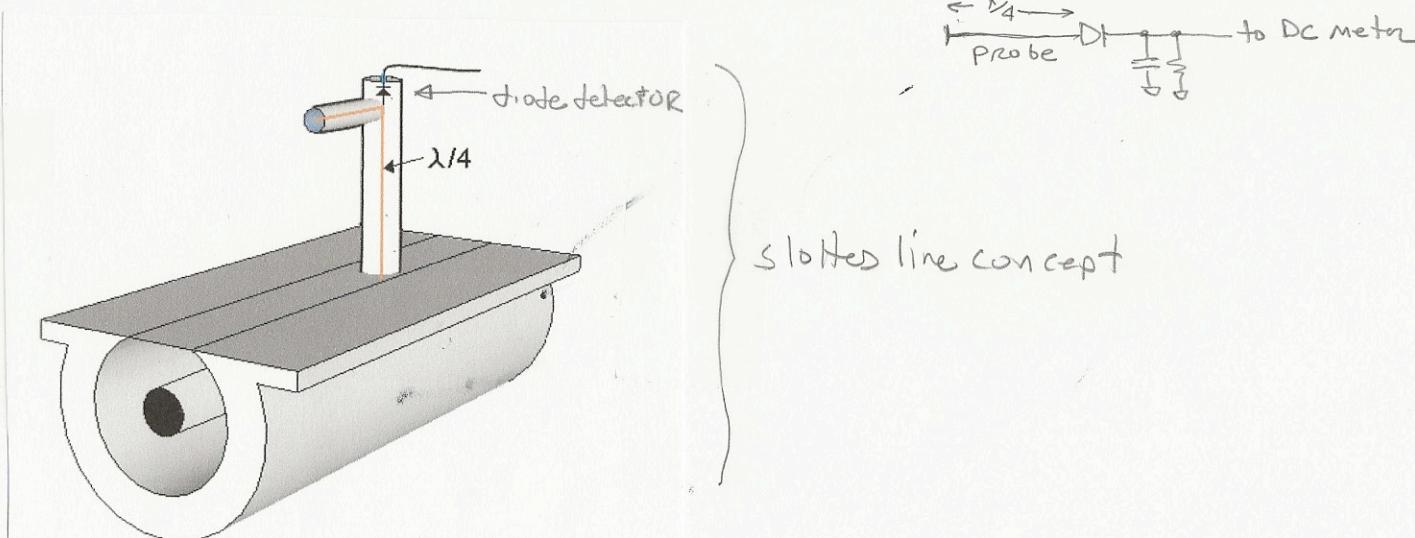
Sometimes we can determine the distance from the load to the first minimum (Z'_{\min}) or the distance to the first maximum (Z'_{\max}) of the standing wave ratio pattern. Then

we could find the phase angle of the reflection coefficient and be able to determine what the load "looks" like.

The instrument ^{uses} to do this is called a slotted line; not often used except in demonstrations or in modest applications.

The slotted line is simply a calibrated length of coaxial line with a movable probe that may be slid up or down the line. The probe slides in a longitudinal line and is close to, but does not touch the inner conductor.

The probe is terminated in a simple diode detector that measures the voltage amplitude of the center conductor against the outer shield.



From: wikipedia.org/wiki/Slotted_line

$z'_{\min} + z'_{\max}$ of Standing Waves

Earlier we saw that ;

$$\begin{aligned} V(z') &= V_0^+ e^{j\beta z'} + V_0^+ \Gamma e^{-j\beta z'} && \text{(lossless)} \\ &= V_0^+ e^{j\beta z'} + V_0^+ |\Gamma| e^{j\theta_L} e^{-j\beta z'} && \text{because } e^a e^b = e^{a+b} \\ &= V_0^+ e^{j\beta z'} \left[1 + |\Gamma| e^{j\theta_L} e^{-2j\beta z'} \right] \\ &= V_0^+ e^{j\beta z'} \left[1 + |\Gamma| e^{j(\theta_L - 2\beta z')} \right] \end{aligned}$$

traveling right traveling left

this could also be considered as $\Gamma(z')$; A generalized voltage reflection coefficient that can be found @ any point z on the line.

Important points :

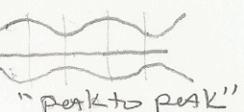
- 1) Magnitude of the reflection coefficient is unchanged across a lossless line
- 2) the phase of the reflection coefficient changes at a rate of 2x the electrical distance from the LOAD.

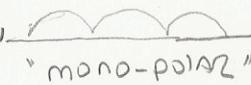
For current, we can likewise say:

$$I(z') = \frac{V_0^+}{Z_0} e^{j\beta z'} \left[1 - |\Gamma| e^{j(\theta_L - 2\beta z')} \right]$$

We are now interested in how $|V(z')|$ varies with distance (z') from the LOAD, especially looking for minimums or maximums.

$|V(z')|$ + $V(z)$ differ. $|V(z')|$ gives us a mono-polar value, $V(z)$ a p-p value

$V(z')$ or 

$|V(z')|$ or 

↑ easier to find min & max!

z_{\min} & z_{\max} of Standing Waves

From before:

$$V(z') = V_0^+ e^{jBz'} \left[1 + |\Gamma_L| e^{j(\Theta_L - 2Bz')} \right]$$

- We would like to have the standing wave plot $|V(z')|$ and find its maximum
- The magnitude of $V(z')$, $|V(z')|$, is $|V(z')| = [V(z')V^*(z')]^{1/2}$
where $V^*(z')$ is the complex conjugate of $V(z')$. We find that: (see next page)
- $|V(z')| = |V_0^+| \underbrace{\left[(1 + |\Gamma_L|)^2 \cos^2(Bz' - \frac{\Theta_L}{2}) + (1 - |\Gamma_L|)^2 \sin^2(Bz' - \frac{\Theta_L}{2}) \right]}_{\text{Maximize this term to}} \quad \text{Find the MAXIMUM of } |V(z')|$
- The maximum will be found where the argument of the \cos^2 function is equal to $n\pi$.
Since $\cos(x)$ is at a maximum at $n\pi$ where $n=0, 1, 2, \dots$, so will \cos^2 .
- So, we can say the maximum standing wave voltages are found from the condition:

$$\boxed{Bz'_{\max} - \frac{\Theta_L}{2} = n\pi ; \quad n=0, 1, 2, 3, \dots}$$
 thus, $z'_{\max} = \frac{n\pi + \frac{\Theta_L}{2}}{B} = \frac{2n\pi + \Theta_L}{2B}$; Also $\Theta_L = z'_{\max} 2B - 2n\pi$
- The voltage minimums will be found $\frac{\lambda}{4}$ away or when:

$$\boxed{Bz'_{\min} - \frac{\Theta_L}{2} = \frac{\pi}{2} + n\pi \quad n=0, 1, 2}$$

$Z_{\min} + Z_{\max}$ of Standing Waves

Being able to find A voltage max or min, including the first one measured from the load,

1. The wavelength of the signal (λ) on the line since points of V_{\max}/V_{\min} occur at $\frac{\lambda}{2}$ intervals. This yields β then AS $\beta = \frac{2\pi}{\lambda}$

2. If we know λ , And thus β , And have A relationship such AS

$$Z'_{\max} = \frac{\theta_L + 2n\pi}{2\beta} \quad \text{we can find } \theta_L, \text{the phase angle of } \Gamma_L \text{ where } \theta_L = Z'_{\max} 2\beta - 2n\pi$$

Δ use θ_L egn.

Then, if we know the SWR, which is related to $|\Gamma_L|$, we can fully characterize a transmission line terminated in an arbitrary load impedance.

$$\text{SWR} = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} \Rightarrow \text{gives } |\Gamma_L|$$

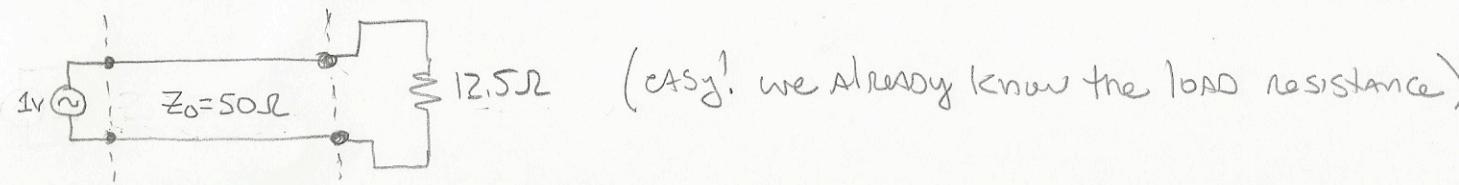
$$Z'_{\max} = \frac{\theta_L + 2n\pi}{2\beta}$$

or

$$\Rightarrow \text{gives } e^{j\theta}$$

$$Z'_{\min} = \frac{\theta_L + (2n+1)\pi}{2\beta}$$

$$|\Gamma_L| e^{j\theta} = \Gamma_L$$



Calculate Γ_L , $|\Gamma_L|$, V_{max} , V_{min} , SWR, Θ_L

sketch $|V(z')|$ as a function of z'/λ

(or $0.6 e^{j180^\circ}$)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{12.5 - 50}{12.5 + 50} = -0.6, \text{ also expressed as } 0.6 e^{j\pi}, \text{ so } \underline{\Gamma_L = -0.6}, \underline{|\Gamma_L| = 0.6}, \underline{\Theta_L = 180^\circ}$$

Solution

$$V_{max} = |V^+| (1 + |\Gamma_L|) = 1 (1 + 0.6) = \underline{1.6V}$$

$$V_{min} = |V^+| (1 - |\Gamma_L|) = 1 (1 - 0.6) = \underline{0.4V}$$

$$\text{SWR} = \frac{|V_{max}|}{|V_{min}|} = \frac{1.6}{0.4} = \underline{4} \quad (\text{A pretty bad match; most transmitters require SWR} < 2)$$

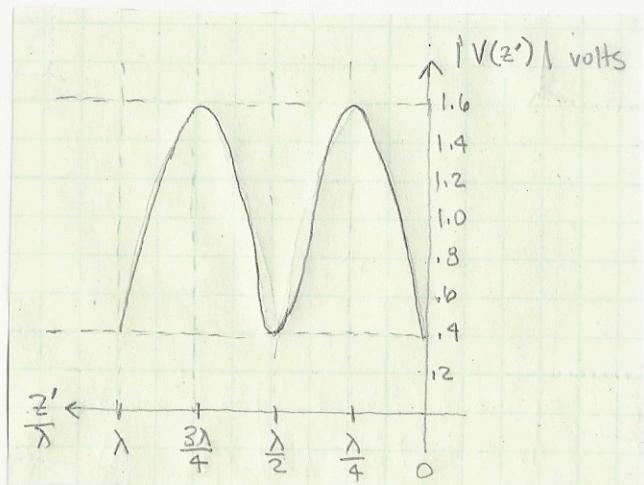
To sketch $|V(z')|$, determine where first maximum is. This is where $\beta z'_{max} - \frac{\Theta_L}{2} = n\pi$ ($n = 0, 1, 2, \dots$)

Closest location is where $n=0$.

Since $\beta = \frac{2\pi}{\lambda}$ and $\Theta_L = \pi$; And $n=0$; $\beta z'_{max} - \frac{\Theta_L}{2} = 0$; or

$$z'_{max} = \frac{\frac{\Theta_L}{2}}{\beta} = \frac{\pi}{2} \cdot \frac{\lambda}{2\pi} = \underline{\frac{\lambda}{4}}$$

Plotting this:



Note: • Resistive loads will show this type of waveform

• if $R_L < Z_0$, $|V(z'=0)|$ will be a minimum
 $R_L > Z_0$, " " " " " maximum

• adjacent min or max values are $\lambda/4$ away

• pattern repeats every $\lambda/2$

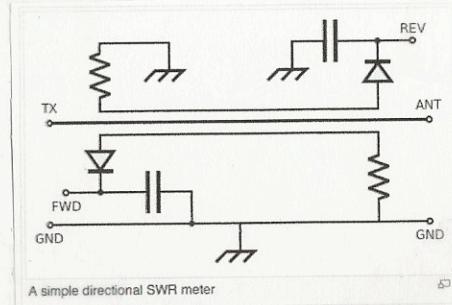
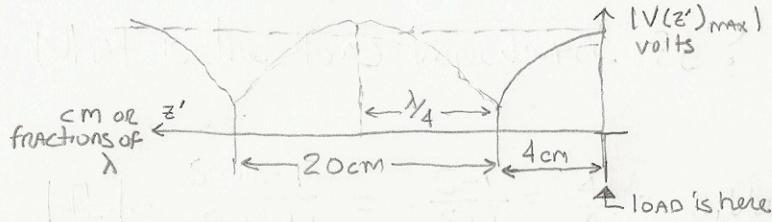
• if $R_L = 0$ min would be at $\text{LOAD} + \frac{\lambda}{4}$ away peak would occur.
 this what we saw before.

A more interesting case:

An SWR measurement on a line reveals an $\text{SWR} = \frac{|V_{\max}|}{|V_{\min}|} = 5$. (this would typically be done with an SWR meter.)

Distance between voltage minima is 20cm.

Distance from termination to nearest voltage minima is 4cm



wikipedia.com/SWR_meter

What is the load impedance Z_L ?

Since minima are 20cm apart, $\lambda/2 = 20\text{cm} + 4\text{cm} = 40\text{cm}$

Thus the distance from the load to the minima (as fraction of λ) is $\frac{4}{40} \text{ or } 0.1\lambda$

So the distance z'_max from the load to the first maxima is 14cm or 0.35λ .

At the first maxima ($n=0$), $\beta z'_\text{max} = \frac{\theta_L}{2}$ or

$$\begin{aligned}\theta_L &= 2\beta z'_\text{max} \\ &= (2)\frac{2\pi}{\lambda}(0.35\lambda) \\ &= 1.4\pi \text{ or } 252^\circ \text{ so } \Gamma_L = e^{j252^\circ} \text{ (3rd quadrant)}\end{aligned}$$

From above $\text{SWR} = \frac{|V_{\max}|}{|V_{\min}|} = 5$, so $\Gamma_L = e^{j252^\circ} = -0.206 - j0.634$

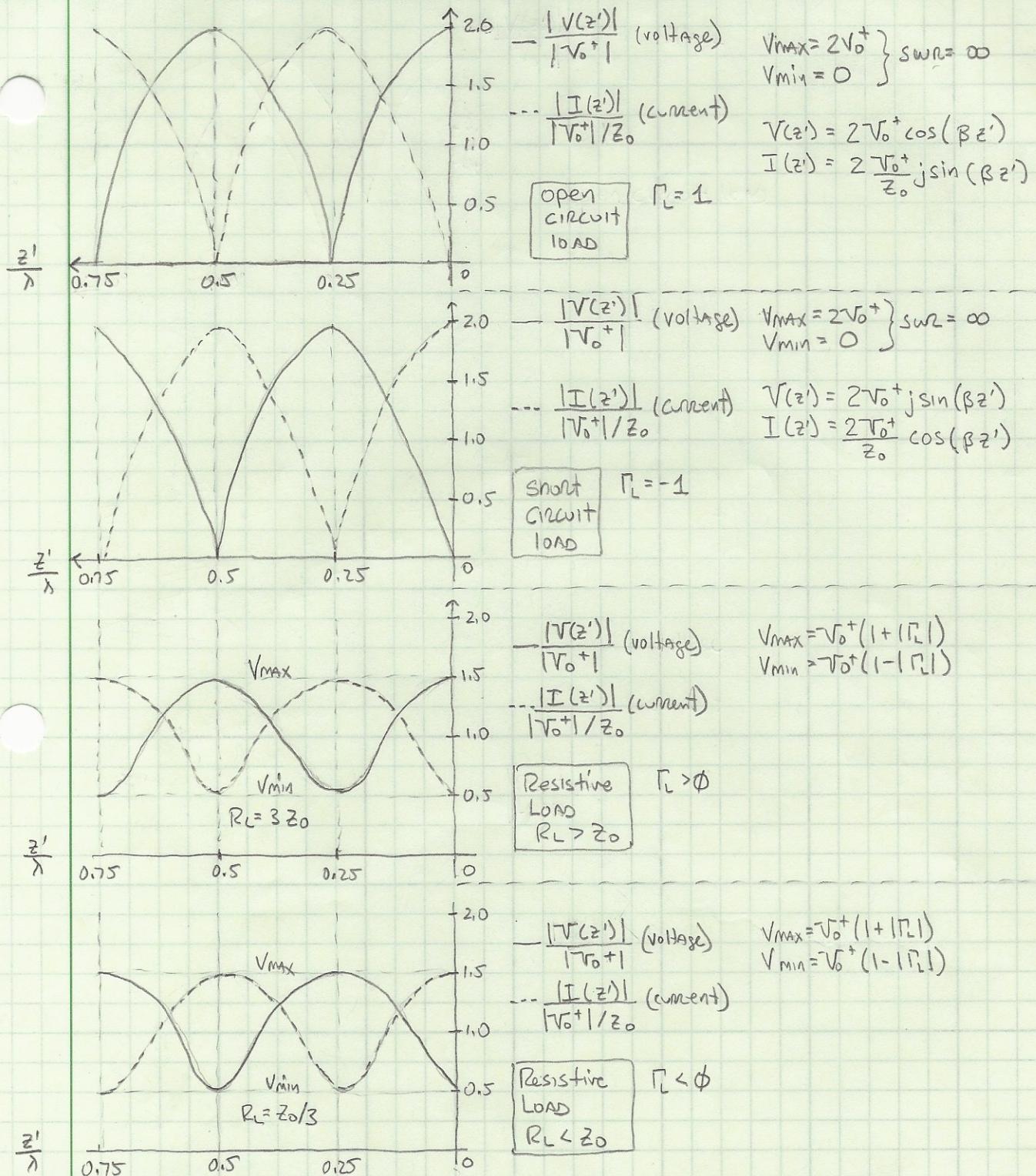
$$|\Gamma_L| = \frac{\text{SWR}-1}{\text{SWR}+1} = \frac{5-1}{5+1} = \frac{4}{6} = 0.66 \implies \text{so } \Gamma_L = 0.66e^{j252^\circ} = -0.206 - j0.634 \text{ (the reflection coefficient has both magnitude + phase unlike a resistive load)}$$

So, finally to find what Z_L is: 14cm (derived from $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$)

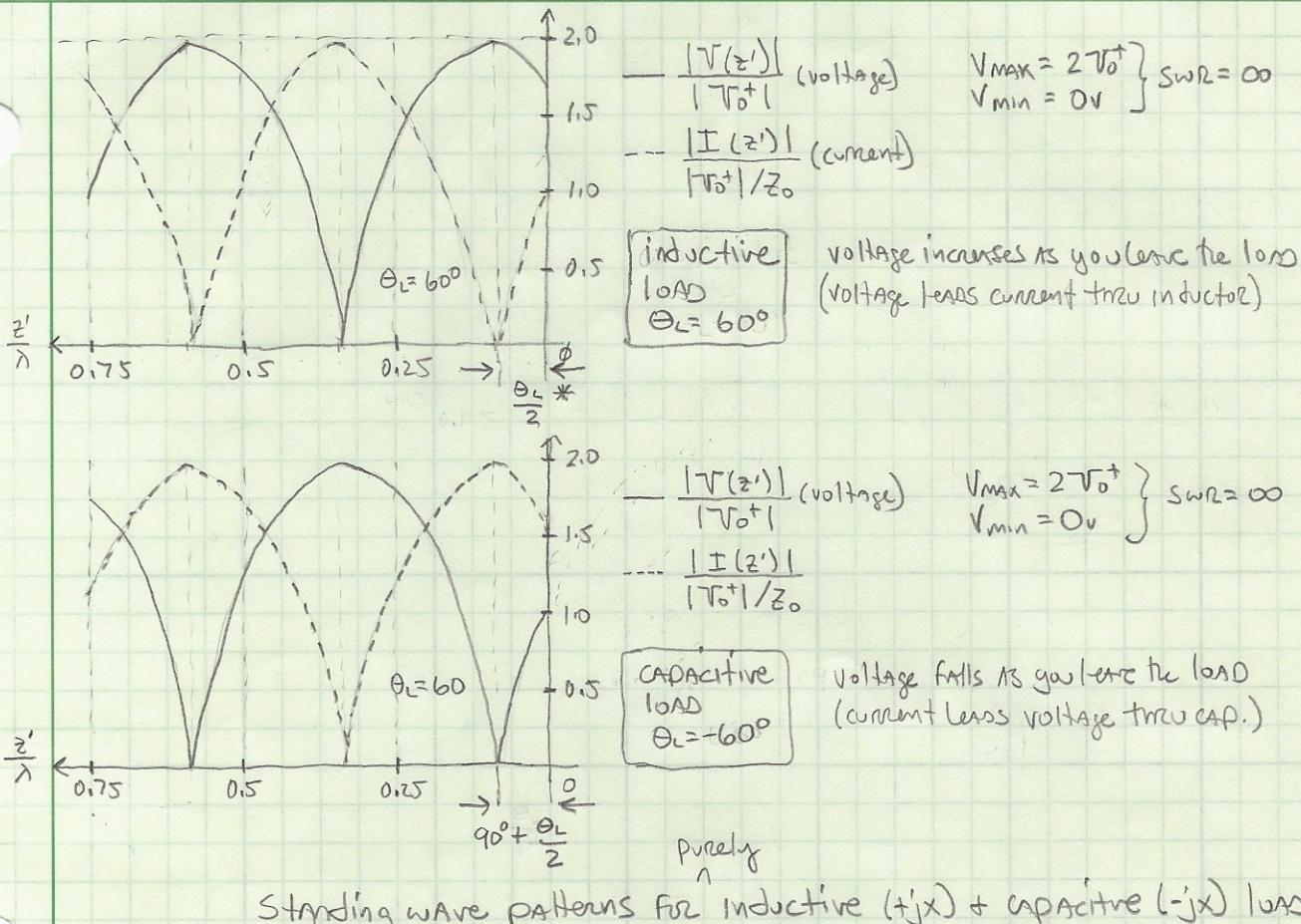
$$Z_L = Z_0 \left(\frac{1 + \Gamma_L}{1 - \Gamma_L} \right) = Z_0 \left(\frac{0.794 - j0.634}{1.206 + j0.634} \right) = Z_0 \left(\frac{1.016 e^{j(-38.6^\circ)}}{1.362 e^{j(27.73^\circ)}} \right) = Z_0 (0.745 e^{j(-66.33^\circ)})$$

this is not to imply that Z_L changes Z_0 . Z_L is fixed!
if Z_0 changes, SWR would change, changing Γ_L
And will give an unchanged Z_L

$$Z_L = Z_0 (0.299 - j0.683)$$



STANDING WAVE PATTERNS FOR OPEN, SHORT AND RESISTIVE TERMINATIONS



Standing wave patterns for inductive ($+jx$) + capacitive ($-jx$) loads

* here's why: $z'_{max} = \frac{\theta_L + 2n\pi}{2\beta}$ and since $\beta = \frac{2\pi}{\lambda}$, at the closest point ($n=0$);

$$z'_{max} = \frac{\theta_L}{2\beta} \Rightarrow 2\beta z'_{max} = \theta_L ; \text{ substitute in for } \beta$$

$$\frac{2\pi z'_{max}}{\lambda} = \frac{\theta_L}{2}$$

location of first maximum

Let's express the fraction as dimensionless
A fraction of 2π RADIANS